How to find Normal probabilities and quantiles using Table B.3.

Given $\mu$ and $\sigma^2$, and that $X \sim N(\mu, \sigma^2)$, to find $P(a \leq X \leq b)$ for given numbers $a$ and $b$.

1. Reexpress the probability in terms of the Standard Normal random variable $Z \sim N(0, 1)$. The applicable change of variables is

$$Z = (X - \mu) / \sigma$$

and the result is

$$P(a \leq X \leq b) = P\left(\frac{a - \mu}{\sigma} \leq X - \mu \leq \frac{b - \mu}{\sigma}\right)$$

$$= P\left(\frac{a - \mu}{\sigma} \leq Z \leq \frac{b - \mu}{\sigma}\right)$$

2. To find the probability in Step 1 extract two values from the Table B.3, $P\left(Z \leq \frac{a - \mu}{\sigma}\right)$ and $P\left(Z \leq \frac{b - \mu}{\sigma}\right)$.

Then the desired probability is found as

$$P\left(a \leq X \leq b\right) = P\left(Z \leq \frac{b - \mu}{\sigma}\right) - P\left(Z \leq \frac{a - \mu}{\sigma}\right).$$

**Example:** $X \sim N(2, 9)$. Find $P(1 \leq X \leq 2)$.

$$P(1 \leq X \leq 2) = P\left(\frac{1 - 2}{3} \leq Z \leq \frac{2 - 2}{3}\right)$$

$$= P\left(-1/3 \leq Z \leq 0\right)$$

$$= P\left(Z \leq 0\right) - P\left(Z \leq -1/3\right)$$

$$= 1/2 - 0.3707$$

$$= 0.1293$$

$$\approx 0.13$$

Finding Normal probabilities is easy, just follow the two-step procedure.

Now consider the inverse problem, namely given a probability in a Normal distribution (a $p$-value), find the corresponding quantile in the Normal population. I.e., given probability $p$ find $x$ such that

$$P(X \leq x) = p$$

Again the procedure is two-step.

1. Reexpress in terms of $Z$. $P(X \leq x) = P\left(Z \leq \frac{x - \mu}{\sigma}\right) \equiv P(Z \leq z) = p$

2. For the given $p$, go to the Table B.3 to find $z$. Solve $z = (x - \mu) / \sigma$ for $x$. 