Large-sample known \( \sigma \) confidence limits for \( \mu \)

\[
\bar{x} \pm z \frac{\sigma}{\sqrt{n}}
\]  
(6.6)

Large-sample confidence limits for \( \mu \)

\[
\bar{x} \pm z \frac{s}{\sqrt{n}}
\]  
(6.9)

Large-sample known \( \sigma \) test statistic for \( \mu \)

\[
Z = \frac{\bar{x} - \#}{\sigma / \sqrt{n}}
\]  
(6.13)

Large-sample test statistic for \( \mu \)

\[
Z = \frac{\bar{x} - \#}{s / \sqrt{n}}
\]  
(6.14)

**Definition 13**

The (Student) \( t \) distribution with degrees of freedom parameter \( \nu \) is a continuous probability distribution with probability density

\[
f(t) = \frac{\Gamma \left( \frac{\nu + 1}{2} \right)}{\Gamma \left( \frac{\nu}{2} \right) \sqrt{\pi \nu}} \left( 1 + \frac{t^2}{\nu} \right)^{-(\nu+1)/2}
\]  
for all \( t \)  
(6.18)

If a random variable has the probability density given by formula (6.18), it is said to have a \( t_{\nu} \) distribution.
Normal distribution confidence limits for $\mu$

\[
\bar{x} \pm t \frac{s}{\sqrt{n}}
\]  

(6.20)

$H_0: \mu = \#$

can be tested using the statistic

Normal distribution test statistic for $\mu$

\[
T = \frac{\bar{x} - \#}{s} \frac{s}{\sqrt{n}}
\]  

(6.21)

and a $t_{n-1}$ reference distribution.

Large-sample confidence limits for $\mu_1 - \mu_2$

\[
\bar{x}_1 - \bar{x}_2 \pm z \sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}}
\]  

(6.30)

$H_0: \mu_1 - \mu_2 = \#$

can be tested using the statistic

Large-sample test statistic for $\mu_1 - \mu_2$

\[
Z = \frac{\bar{x}_1 - \bar{x}_2 - \#}{\sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}}}
\]  

(6.31)
If \( r \) numerical samples of respective sizes \( n_1, n_2, \ldots, n_r \) produce sample variances \( s_1^2, s_2^2, \ldots, s_r^2 \), the pooled sample variance, \( s_p^2 \), is the weighted average of the sample variances, where the weights are the sample sizes minus 1. That is,

\[
s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \cdots + (n_r - 1)s_r^2}{(n_1 - 1) + (n_2 - 1) + \cdots + (n_r - 1)} \tag{7.7}
\]

The pooled sample standard deviation, \( s_p \), is the square root of \( s_p^2 \).

**Normal distributions**

\( (\sigma_1 = \sigma_2) \) confidence limits for \( \mu_1 - \mu_2 \)

\[
\bar{x}_1 - \bar{x}_2 \pm t_{sp} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \tag{6.35}
\]

where \( t \) is chosen such that the probability that the \( t_{n_1+n_2-2} \) distribution assigns to

\( H_0: \mu_1 - \mu_2 = \# \)

can be tested using the statistic

**Normal distributions**

\( (\sigma_1 = \sigma_2) \) test statistic for \( \mu_1 - \mu_2 \)

\[
T = \frac{\bar{x}_1 - \bar{x}_2 - \#}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \tag{6.36}
\]

and a \( t_{n_1+n_2-2} \) reference distribution.

**Satterthwaite's "estimated degrees of freedom"**

\[
\hat{\nu} = \frac{\left( \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{\frac{(n_1 - 1)s_1^4}{n_1^2} + \frac{(n_2 - 1)s_2^4}{n_2^2}} \tag{6.37}
\]

**Satterthwaite (approximate) normal distribution confidence limits for \( \mu_1 - \mu_2 \)**

\[
\bar{x}_1 - \bar{x}_2 \pm t_{\hat{\nu}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \tag{6.38}
\]
Figure 6.16 $\chi^2$ probability densities for $\nu = 1, 2, 3, 5, \text{ and } 8$

**Definition 15**

The $\chi^2$ (Chi-squared) distribution with degrees of freedom parameter, $\nu$, is a continuous probability distribution with probability density

$$f(x) = \begin{cases} \frac{1}{2^{\nu/2} \Gamma\left(\frac{\nu}{2}\right)} x^{(\nu/2)-1} e^{-x/2} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases} \tag{6.39}$$

If a random variable has the probability density given by formula (6.39), it is said to have the $\chi^2_\nu$ distribution.

Normal distribution confidence limits for $\sigma^2$

$$\frac{(n-1)s^2}{U} \text{ and } \frac{(n-1)s^2}{L} \tag{6.42}$$

where $L$ and $U$ are such that the $\chi^2_{n-1}$ probability assigned to the interval $(L, U)$,

$$H_0: \sigma^2 = \#$$

can be tested using the statistic

Normal distribution test statistic for $\sigma^2$

$$X^2 = \frac{(n-1)s^2}{\#} \tag{6.43}$$

and a $\chi^2_{n-1}$ reference distribution.
Definition 16

The (Snedecor) $F$ distribution with numerator and denominator degrees of freedom parameters $v_1$ and $v_2$ is a continuous probability distribution with probability density

$$f(x) = \begin{cases} \frac{\Gamma \left( \frac{v_1 + v_2}{2} \right) \left( \frac{v_1}{v_2} \right)^{v_1/2} x^{v_1/2 - 1}}{\Gamma \left( \frac{v_1}{2} \right) \Gamma \left( \frac{v_2}{2} \right) \left( 1 + \frac{v_1 x}{v_2} \right)^{(v_1 + v_2)/2}} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (6.44)$$

where $L$ and $U$ are ($F_{v_1-1,v_2-1}$ quantiles)

$$H_0: \frac{\sigma_1^2}{\sigma_2^2} = \#$$

can be tested using the statistic

$$F = \frac{s_1^2 / s_2^2}{\#} \quad (6.47)$$

and an $F_{v_1-1,v_2-1}$ reference distribution.
linear combination of population means

\[ L = c_1 \mu_1 + c_2 \mu_2 + \cdots + c_r \mu_r \]  \hspace{1cm} (7.16)

linear combination of sample means

\[ \hat{L} = c_1 \bar{y}_1 + c_2 \bar{y}_2 + \cdots + c_r \bar{y}_r \]  \hspace{1cm} (7.17)

\[ Z = \frac{\hat{L} - E \hat{L}}{\sqrt{\text{Var} \hat{L}}} = \frac{\hat{L} - L}{\sigma \sqrt{\frac{c_1^2}{n_1} + \frac{c_2^2}{n_2} + \cdots + \frac{c_r^2}{n_r}}} \]  \hspace{1cm} (7.18)

\[ T = \frac{\hat{L} - L}{s_p \sqrt{\frac{c_1^2}{n_1} + \frac{c_2^2}{n_2} + \cdots + \frac{c_r^2}{n_r}}} \]  \hspace{1cm} (7.19)

instead. The fact is that under the current assumptions, the variable (7.19) has a \( t_{n-r} \) distribution. And this leads in the standard way to the fact that the interval with endpoints

Confidence limits for a linear combination of means

\[ \hat{L} \pm t_{SP} \sqrt{\frac{c_1^2}{n_1} + \frac{c_2^2}{n_2} + \cdots + \frac{c_r^2}{n_r}} \]  \hspace{1cm} (7.20)

P-R two-sided simultaneous 95% confidence limits for \( r \) means

\[ \bar{y}_i \pm k^*_2 \frac{s_p}{\sqrt{n_i}} \]  \hspace{1cm} (7.28)

Tukey's two-sided simultaneous confidence limits for all differences in \( r \) means

\[ \bar{y}_i - \bar{y}_{i'} \pm \frac{q^*}{\sqrt{2}} s_p \sqrt{\frac{1}{n_i} + \frac{1}{n_{i'}}} \]  \hspace{1cm} (7.36)
For any $n$ numbers $y_{ij}$

$$
(n - 1)s^2 = \sum_{i=1}^{r} n_i (\bar{y}_i - \bar{y})^2 + (n - r)s_p^2 
$$

(7.49)

or in other symbols,

$$
\sum_{i,j} (y_{ij} - \bar{y})^2 = \sum_{i=1}^{r} n_i (\bar{y}_i - \bar{y})^2 + \sum_{i=1}^{r} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_i)^2
$$

(7.50)

**Definition 4**

In a multisample study, $(n - 1)s^2$, the sum of squared differences between the raw data values and the grand sample mean, will be called the **total sum of squares** and denoted as $SSTot$.

**Definition 5**

In an unstructured multisample study, $\sum n_i (\bar{y}_i - \bar{y})^2$ will be called the **treatment sum of squares** and denoted as $SSTR$.

**Definition 6**

In a multisample study, the sum of squared residuals, $\sum (y - \bar{y})^2$ (which is $(n - r)s_p^2$ in the unstructured situation) will be called the **error sum of squares** and denoted as $SSE$.

$$
SSTot = SSTR + SSE
$$

(7.51)

Table 7.12

General Form of the One-Way ANOVA Table

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>$SSTR$</td>
<td>$r - 1$</td>
<td>$SSTR/(r - 1)$</td>
<td>$MSTR/MSE$</td>
</tr>
<tr>
<td>Error</td>
<td>$SSE$</td>
<td>$n - r$</td>
<td>$SSE/(n - r)$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$SSTot$</td>
<td>$n - 1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>