Outlines of Proofs of Theorems 3 and 2 (of the Stat 643 Summary)

Outline of the proof of Theorem 3 (using Theorem 2):

\[(\implies)\]
\[
\frac{dP_\theta}{d\mu} = \cdot \frac{dP_\theta}{d\lambda} \cdot \frac{d\lambda}{d\mu} \text{ plus a.e.'s}
\]
for \(\frac{dP_\theta}{d\lambda} = g_\theta \circ T \) (Theorem 2) and \(\frac{d\lambda}{d\mu} = h\)

\[(\iff)\]
\[
\frac{dP_\theta}{d\mu} = \cdot \frac{dP_\theta}{d\lambda} \cdot \frac{d\lambda}{d\mu} \text{ plus a.e.'s}
\]
for \(\frac{dP_\theta}{d\mu} = (g_\theta \circ T) \cdot h \) and \(\frac{d\lambda}{d\mu} = h \cdot \sum_{i=1}^{\infty} c_i (g_{\theta_i} \circ T)\)

which implies that \(\frac{dP_\theta}{d\lambda}\) is of the “right” form

Outline of the proof of Theorem 2 (using Theorem 1):

\[(\implies)\]
\[P^0_\theta \text{ and } \lambda^0 \text{ the restrictions of } P_\theta \text{ and } \lambda \text{ to } \mathcal{B}_0\]
\[
\frac{dP_\theta^0}{d\lambda^0} = g_\theta \circ T \text{ by Lehmann’s Theorem}
\]
\[\text{show } g_\theta \circ T = \frac{dP_\theta}{d\lambda} \text{ using facts about } P(B|\mathcal{B}_0)\]

\[(\iff)\]
\[\text{Show } E_\theta (I_B|\mathcal{B}_0) = E_\lambda (I_B|\mathcal{B}_0) \text{ using the representation of } \frac{dP_\theta}{d\lambda}\]