1-sided (single limit) cases: (single lower limit $L$)

$$P = \Phi\left(\frac{L - \mu}{\sigma}\right)$$

depending upon set-up, either a function of $\mu$ only or a function of both $\mu$ and $\sigma$

small $\mu$ makes $P$ big — small $\sigma$ suggests that $\mu$ is small — so we should reject for small $\sigma$ i.e.

reject 1st if $\bar{X} < k$
accept 1st if $\bar{X} > k$

$$p(\mu) = \Phi\left(\frac{L - \mu}{\sigma}\right) \quad \text{(a function of $\mu$ alone)}$$

$$P_{a}(\mu) = P[\bar{X} > k] = 1 - P[\bar{X} < k]$$

a function of $\mu$ alone $\Rightarrow$ $1 - \Phi\left(\frac{k - \mu}{\sigma/\sqrt{n}}\right)$

2-sided (double limits) cases:

$$P = 1 - \left[\Phi\left(\frac{U - \mu}{\sigma}\right) - \Phi\left(\frac{L - \mu}{\sigma}\right)\right]$$

generally, depending upon scenario, this is either a function of $\mu$ alone, or of $\mu$ and $\sigma$

Single Limit "$\mu$ known" cases (lower limit version)

So as I vary $\mu$

$$(p(\mu), Pa(\mu))$$

trays out a path in $(P, Pa)$-plane that I can use as an OC-curve.

See VIJ section 8.2 for classical 2-point design problem (choosing $n, k$ to get OC-curve to pass through 2 points of interest)

Double Limit "$\mu$ known" cases:

$m$ far from $\frac{U + L}{2}$ makes $P$ big — therefore one might reject $a$ lot if $\bar{X}$ is too far from $\frac{U + L}{2}$.
\[
\begin{align*}
\text{accept lot if } & \frac{U_L + k}{2} - k < \bar{z} < \frac{U_H + k}{2} + k \\
\text{reject lot otherwise} & \\
\Pr(\mu) = 1 - \left[ \Phi \left( \frac{U_H - \mu}{\sigma} \right) - \Phi \left( \frac{U_L - \mu}{\sigma} \right) \right] & \\
\Pr(\mu) & = \phi \left( \frac{\frac{U_H + k}{2} - \mu}{\sigma/\sqrt{n}} \right) - \phi \left( \frac{\frac{U_L + k}{2} - \mu}{\sigma/\sqrt{n}} \right) \\
\text{and now there are } & \\
2 \mu \text{s giving each } & \\
\begin{array}{c}
\text{value of } \mu \text{ p is minimum at } \frac{L + U}{2} \\
\text{traces out a path in the } (p, Pa) \text{- plane} \\
\end{array} & \\
\text{that can serve as an O-C curve (both } & \\
\mu \text{s giving a particular } p \text{ give the same } Pa) & \\
\text{that can serve as an O-C curve (both } & \\
\mu \text{s giving a particular } p \text{ give the same } Pa) & \\
\text{See } V+J \text{ for classical (2pt) design} & \\
\text{(choice of } L \text{ and } n) & \\
\text{Single Limit "T unknown" Cases : (lower limit)} & \\
\text{note that } p \text{ depends upon } (\mu, \sigma) & \\
\text{and } \bar{z} \text{-value associated with } L & \\
\frac{L - \mu}{\sigma} & \\
\text{when this is small (large negatively)} & \\
\text{all is well i.e. } p \text{ is small} & \\
\text{so a plausible} & \\
\end{align*}
\]
\[
\begin{align*}
&= P\left[ \frac{X - \mu}{\sigma/\sqrt{n}} - \frac{L - \mu}{\sigma/\sqrt{n}} > k \right] \\
&= P\left[ \frac{n(N(0,1)) - L - \mu}{\sigma/\sqrt{n}} > \sigma \sqrt{n} k \right] \\
&= P\left[ \text{a noncentral } t_{n-1}(\mu_k) \text{ r.v.} > \sqrt{n} k \right]
\end{align*}
\]

with mean \( \mu_k \) and variance \( \sigma^2 \left( \frac{1}{n} + \frac{k^2}{2n} \right) \)

Note that \( \text{Vars} = \bar{X}^2 \sigma^2(\frac{n}{n-1}) \)

also \( \text{Vars} \approx \left( \frac{1}{2\sqrt{\bar{X}^2 \sigma^2}} \right)^2 \text{Vars}^2 \)

\[
\approx 1 - \Phi \left( \frac{L - (M - k\sigma)}{\sqrt{\frac{1}{n} + \frac{k^2}{2n}}} \right)
\]

and using this is close to the Wallis approximation.
A common choice for \( \theta \) is the \( \text{UNIVUE} \)
of \( \hat{P} \). This was used, e.g., in the Bayesian
M.L.S.A. 4+ variables, each over
all pictures for each plan. Usually
avoided. Plans = each plan, usually
4+ variables. Feedback form.

\[ \text{Reject if } \hat{P} > k \]

An estimator of \( \hat{P} \) is usually
\( \hat{P} \). say \( \hat{P}(z) \).

\[ \text{some estimators of } \hat{P} \]

\[ \text{Double limits on unknown cases} \]

\[ \text{This is a messy problem. We clean up} \]
\[ \text{all (or most) of the acceptance regions.} \]

\[ \text{Do some} \]

\[ \text{in the (215) plane} \]

...
for the best of these. This band may not be so wide... perhaps thin enough to ignore this issue.

3) besides, it's not so clear to Vardeman that treating at \( \mu \hat{\mu} \) with a given \( \hat{\mu} \) equally is rational.

4) This whole enterprise (variables acceptance sampling) comes unglued if I admit any possibility of measurement error.

Philosophy/Caveats

1) in comparison to corresponding attributes plans, variables acceptance sampling plans typically provide huge sample size savings—this is particularly true when my concern is small \( p \).

2) but... This shift depends critically on the parametric model assumptions (particularly on the shape of the extreme tails of a distribution) that can only be checked with a huge sample size.

3) \( X \sim N(\mu, \sigma^2) \) den of the "real" variation in population:

\[
y = x + \epsilon \\
\epsilon \sim N(0, \sigma^2) \\
\epsilon \sim N(0, \sigma^2) \quad \text{independent}
\]

I end up with \( \bar{y} \) and \( s_y \) (not \( \bar{x} \), \( s_x \))—as long as \( \chi^2 > 0 \) there will be \( \sigma^2 \) small enough so that population variability hides in measurement noise.