Attention: Henceforth computing turned in as part of Stat 511 HW must be done in R.
Becoming familiar with R is a course objective, and besides I don't want Norma to have to try to decipher code from some other system.

1. In class Vardeman claimed that that
\[ \{ \zeta \mid \zeta \text{ is "special"} \} \equiv \{ \zeta \mid d \in C(X')^\perp \text{ then } d \perp \zeta \} = C(X') \]
Prove this. (Argue that \( \zeta \in C(X') \Rightarrow \zeta \) is "special", and then that \( \zeta \) is "special" \( \Rightarrow \zeta \in C(X') \). For the second of these, write \( \zeta = P_{X'\zeta} + (I - P_{X'\zeta})\zeta \) and note that if \( \zeta \in C(X') \) then \( d^\perp = (I - P_{X'\zeta})\zeta \) is a nonzero element of \( C(X')^\perp \). Consider \( \zeta'd^\perp \).)

2. In the context of Christensen's Example 1.0.2 and the fake data vector used in Problem 6 of HW1, use R and weighted generalized least squares to find appropriate estimates for
\[
\begin{bmatrix}
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1
\end{bmatrix}
\]
in the Aitken models with

\[
V_1 = \text{diag}(1,1,4,1,4,4) \quad \text{and} \quad V_2 = \begin{bmatrix}
1 & .5 & .5 & 0 & 0 & 0 \\
.5 & 1 & .5 & 0 & 0 & 0 \\
.5 & .5 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & .5 \\
0 & 0 & 0 & 0 & .5 & 1
\end{bmatrix}
\]

For the first of these covariance structures, compare the (Aitken model) covariance matrices for generalized least squares estimators to (Aitken model) covariance matrices for the OLS estimators of \( E \) and the \( C^B \) above.

3. Continue in the context of Christensen's Example 1.0.2. For a given \( \sigma^2 \), how do the covariance matrices for BLUEs of
\[
\begin{bmatrix}
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1
\end{bmatrix}
\]
compare for the Gauss Markov model and in the Aitken model with \( \text{Cov} \xi = V_2 \) for \( V_2 \) as in Problem 2? (I'm talking about the Gauss-Markov covariance matrix of the Gauss-Markov BLUE versus the Aitken covariance matrix of the Aitken BLUE.) In retrospect, why is this sensible in light of the nature of \( V_2 \)?

4. Consider the (non-full-rank) "effects model" for the \( 2 \times 2 \) factorial (with 2 observations per cell) called "Example d)" in the first lecture. Determine which of the parametric functions below are estimable, and for each that is, identify a vector \( \theta \) such that \( E \theta'Y \) is equal to that function.
\[
\mu, \alpha_1, \mu + \alpha_1, \alpha_1 - \alpha_2, \mu + \alpha_1 + \beta_1, \mu + \alpha_1 + \beta_1 + \alpha_{\beta_{11}}, \alpha_{\beta_{11}}, \alpha_{\beta_{12}} - \alpha_{\beta_{11}} - (\alpha_{\beta_{22}} - \alpha_{\beta_{21}})
\]