1. Use (mixed model) thermocouple calibration problem of HW8.

(a) Results match those obtained in 3(e) of HW8. The ML estimates are: $\hat{\sigma}_\gamma^2 = 0.0629$, $\hat{\sigma}_\delta^2 = 0.4441$ and $\hat{\sigma}^2 = 0.3733$.

(b) The REML estimates are: $\hat{\sigma}_\gamma^2 = 0.3438$, $\hat{\sigma}_\delta^2 = 0.8087$, and $\hat{\sigma}^2 = 0.2848$. REML estimates for $\sigma_\gamma^2$ and $\sigma_\delta^2$ are bigger than ML estimates.

(c) $C_{\hat{\beta}_{MLE}} = C_{\hat{\beta}_{REMLE}} = (\hat{\alpha} = 99.4551, \hat{\beta} = 9.7936)$

(d) $\hat{u}_{MLE} = -0.02172176 0.1511162 -0.1293945 -0.6478563 -0.2366669 0.8845232$
$\hat{u}_{REMLE} = -0.02999012 0.4881394 -0.4581493 -0.6637030 -0.3404386 1.0041420$

(f) $\text{Var} = G - G\%*t(Z)\%*P\%*Z\%*G; \text{stderr} = \text{sqrt(diag(Var))}$

(g) $\text{Var} = \text{sqrt(diag(Var))}$
<table>
<thead>
<tr>
<th></th>
<th>MLE</th>
<th>std error</th>
<th>REMLE</th>
<th>std error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>99.4334</td>
<td>0.2632</td>
<td>99.4251</td>
<td>0.3117</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>9.1457</td>
<td>0.3819</td>
<td>9.1299</td>
<td>0.5128</td>
</tr>
</tbody>
</table>

2. ` thermo <- read.table("hw09.txt", header=T)`
   
   ` gd <- groupedData(y~temp|group, data=thermo)`
   
   ` fm1 <- lme(y~temp, data=gd, random="1+temp|group, method="ML")`
   
   ` fm2 <- lme(y~temp, data=gd, random="1+temp|group, method="REML")`
   
   ` fm3 <- update(fm1, random=pdDiag(~1+temp), method="ML")`
   
   ` fm4 <- update(fm2, random=pdDiag(~1+temp), method="REML")`

   ```r
   > fixed.effects(fm3)  # ML results in 1(c)
   (Intercept)  temp
   99.45513 9.79359
   > random.effects(fm3)  # ML results in 1(e)
   (Intercept)  temp
   1 -0.02181365 -0.6478858
   2 0.15202592 -0.2369421
   3 -0.13021227 0.8848279
   > coefficients(fm3)
   (Intercept)  temp
   1 99.43331 9.145704  # This row was obtained in 1(g) using ML
   2 99.60715 9.556648
   3 99.32492 10.678418
   ```

   In the function `intervals()` the point estimates for the variance components correspond to the square root of the estimates obtained in 1(a). (e.g., $\text{sd}((\text{Intercept})) = \sqrt{\hat{\sigma}^2} = \sqrt{0.0629} \approx 0.2516$). Note that given the size of the data, the confidence intervals for the ML estimates of the variance components are very wide.

   ```r
   > intervals(fm3)
   Approximate 95% confidence intervals
     Fixed effects:
     lower  est.  upper
     (Intercept) 98.632658 99.45513 100.27760
     temp  8.757383 9.79359 10.82980
     Random Effects:
     Level: group
     lower  est.  upper
     sd((Intercept)) 4.895553e-05 0.2516244 1293.313462
     sd(temp) 2.272768e-01 0.6666714 1.955548
     Within-group standard error:
     lower  est.  upper
     0.1449576 0.6106299 2.5722616
   ```

   Results for REML estimates are obtained using the same functions with the model stored in `fm4`. 