Stabilization of Slotted ALOHA Spread-Spectrum Communication Networks

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Abstract—This paper presents a method for stabilizing slotted ALOHA frequency-hop communication networks by controlling code rate. In particular, we consider a fully connected network in which a finite number of transmitter-receiver pairs exchanges packets of information encoded by Reed-Solomon codes. A principle of flow balance to stability analysis is adopted, and it is shown that network stability can be ensured by controlling code rate. The requirement on code rate for ensuring network stability is examined. In addition, the mean delay and the mean channel throughput of the network are discussed.

I. INTRODUCTION

In recent years, spread spectrum multiple access (SSMA) has been receiving an increasing amount of attention as a useful technique for transmitting many user's data simultaneously [1], [2]. SSMA transmissions offer a number of advantages such as immunity to interference and jamming and low probability of interception, along with uncoordinated multiple access capability. The use of error-control coding in SSMA communication systems has been shown to be especially effective in increasing the multiple-access capability [1], [3]. The two most common forms of SSMA are frequency-hopped (FH) SSMA and direct-sequence (DS) SSMA. This paper is concerned with a method for stabilizing FH-SSMA communication systems.

When many users try to transmit bursty packets in an uncoordinated fashion (ALOHA-protocol [6]), it is possible for packets to encounter traffic dependent “collisions,” in conventional random access systems because there is no control on the number of simultaneous transmissions. This indicates that spread-spectrum packet radio networks will display the same qualitative bistability and saturation effects that characterize the conventional ALOHA systems, (which can be thought of as a degenerate special case of SSMA system). Our objective here is to stabilize frequency-hop packet radio networks by noting that such instability occurs due to the packet collisions (i.e., unsuccessful transmissions), and the rate of unsuccessful transmissions can be controlled by controlling the code rate being used.

We assume that each packet consists of an \((n, k)\) Reed-Solomon code, and erasure-only decoding is employed.

A symbol erasure takes place for a particular transmitter either if one or more other transmitters transmit in the same frequency slot at the same time (an event known as "hit") or if the background noise level is high enough. We let \(P_h\) be the conditional probability of a symbol being erased given that it is not hit, and will examine how the system performance changes as \(P_h\) changes. We will consider the situation in which the demodulator can determine the hit. We call this the case of side information available. Several methods of generating side information are suggested in [5], [16]. In our model of a frequency-hop radio network, there is a band of \(q\) frequency slots available and each transmitter-receiver pair has a frequency hopping pattern that randomly hops among all \(q\) frequency slots with probability \(1/q\) for each slot, independent of the previous hop frequencies (i.i.d. hopping).

A packet erasure (i.e., unsuccessful transmission of a packet) takes place when more than \((n-k)\) symbols are erased in a packet. In this event, the transmitter must retransmit.

Up to this point, most of the literature has dealt with multiple-access protocols and SSMA in a separate manner. An exception to this has been the paper by Hajek [7] where a recursive retransmission control policy with a frequency hopped signal has been discussed. Stabilization of random access code-division multiple-access systems by controlling retransmission delay has been examined in [4]. In [8], [9] various retransmission control methods have been proposed to stabilize the ALOHA-type system that does not employ spread-spectrum techniques.

II. SYSTEM MODEL

We consider a slotted ALOHA spread-spectrum packet radio network with \(K\) transmitter-receiver pairs (also called user pairs), each pair sharing knowledge of a unique frequency hopping (FH) pattern. Transmission of data is done in packets, each packet being of length \(n\) symbols (each symbol transmitted in the duration of a hop, or dwell time), and the same error-control code is used by all user pairs.

The packet flow in an ALOHA-SSMA system is shown in Fig. 1. The channel input consists of new packets and retransmitted packets. A fraction of these are received successfully, while those which are received in error are eventually retransmitted. The number of attempted trans-
missions in a time slot is denoted by $I$, and its probability mass function is denoted by $f_j(i)$.

We assume that each receiver is able to hear the transmission of each of the $J$ transmitters. Each receiver does not attempt to dehop and demodulate signals other than its own. The signal of each user is, however, present in the front end of each receiver and is a potential source of interference. An example of this is a satellite multiple-access broadcast system or a fully connected network. Thus, the interference traffic level will be the same at each receiver in the network, and we assume that exactly the same code is used for all user pairs in the network.

For simplicity of analysis, we often modify the model in Fig. 1 as in Fig. 2. This kind of modification in the performance analysis of ALOHA-type systems was first used by Kobayashi et al. [10]. The purpose of the modification is to merge the two inputs (from $T$ and $RT$ modes) to the slotted ALOHA channel into one. Since we usually assume bursty users, we now consider the modification under the condition of $\alpha \leq \beta$.

Let $U_1$ and $W_1$ be random variables representing the intervals (number of time slots) during which a user is in $T$ and $RT$ modes, respectively. The random variables $U_1$ and $W_1$ are geometrically distributed with means $1/\alpha$ and $1/\beta$, respectively. Since $1/\alpha > 1/\beta$, we can decompose $U_1$ into two components in such a way that $U_1 = V + W_1$, where $V$ and $W_1$ are statistically independent. Then it is easily shown that Prob $[V = 0] = \alpha/\beta$ and Prob $[V \geq 1] = 1 - \alpha/\beta$ and that the conditional distribution of $V$, given that $V \geq 1$, is geometric with mean $1/\alpha$. From this result, we can get a modified model as shown in Fig. 2. It is apparent in Fig. 2 that $T$ mode in Fig. 1 has been decomposed into two tandem phases, $TH$ and $TR$ modes, and that $RT$ mode in Fig. 1 has become a part of $TR$ mode.

The equivalence of the two models can be shown as follows. Let $W_2$ be random variables representing the intervals (number of slots) during which a user is in $TR$ mode, and $U_2$ be a random variable representing the interval (number of slots) from the moment a user enters either $TH$ mode or $TR$ mode until the instant the user moves out from $TR$ mode. (Note that $U_2$ corresponds to $U_1$.) In addition, we denote the probability generating functions of $U_1$, $U_2$, $W_1$, $W_2$ as $G_{U_1}(z)$, $G_{U_2}(z)$, $G_{W_1}(z)$, $G_{W_2}(z)$, respectively. Then, we easily have

$$G_{U_1}(z) = \frac{\alpha z}{1 - (1 - \alpha)z}$$

and

$$G_{U_2}(z) = \frac{p z}{1 - (1 - p)z},$$

since $U_1$ and $W_1$ are geometrically distributed with means $1/\alpha$ and $1/\beta$, respectively. Similarly, it is apparent that

$$G_{W_2}(z) = \frac{p z}{1 - (1 - \alpha)z}.$$  

From Fig. 2 we also have

$$G_{U_2}(z) = \frac{1 - \alpha/p}{\alpha z/(1 - (1 - \alpha)z)} G_{W_2}(z) + (\alpha/p) G_{W_2}(z)$$

$$= \frac{\alpha z}{1 - (1 - \alpha)z}.$$  

Thus, $U_1$ and $W_2$ have the same probability distributions as those of $U_1$ and $W_1$, respectively. Therefore there is no performance difference between these two models. If we let $N$ be a random variable representing the number of users in $TR$ mode, then $f_k(i)$ is given by

$$f(i) = \binom{N}{i} p^i (1 - p)^{N-i}.  \tag{1}$$

III. PACKET ERASURE PROBABILITY

The packet erasure probability (i.e., the probability of unsuccessful transmission) with $i$ simultaneous transmissions, denoted as $P_e(i)$, is given by

$$P_e(i) = \sum_{j=i-k+1}^n \binom{n}{j} P_{er,i}^j (1 - P_{er,i})^{n-j} \tag{2}$$

where

$$P_{er,i} = 1 - (1 - P_b)^{i-1} + P_b (1 - P_b)^{i-1}  \tag{3}$$

is the probability of symbol erasure with $i$ simultaneous transmissions. $P_0$ is the probability of symbol erasure given that the symbol is not hit (occurring due to the background noise), and $P_b$ is the probability of two symbols being hit and is given in [11]

$$P_b = 2/q - 1/q^2  \tag{4}$$
where we have assumed the asynchronous frequency hopping situation.

It has been shown in Kim and Stark [3] that as both \( n \) and \( k \) become large with rate \( r = k/n \) held constant,

\[
\lim_{n,k} P_E(i) = \begin{cases} 
0, & r < (1 - P_0) (1 - P_h)^{i-1} \\
0.5, & r = (1 - P_0) (1 - P_h)^{i-1} \\
1, & r > (1 - P_0) (1 - P_h)^{i-1} 
\end{cases}
\]  

(5)

That is, error-free communication is asymptotically possible with \( i \) simultaneous transmissions if

\[
i < 1 + \frac{\log(r/(1 - P_0))}{\log(1 - P_h)} = K_m.
\]  

(6)

IV. STABILITY AND ERROR CONTROL

In this section, we use the principle of flow balance to study the dynamics of ALOHA-SSMA system. We will be able to examine the stability and the mean channel throughput without going through extensive computations.

The basic idea of the flow balance principle is as follows [8], [12]. Suppose that the system is at state \( N \), and let \( S_m(N) \) and \( S_out(N) \) be, respectively, the mean number of net packet flows into the system and the mean number of packet flows out of the system in a cycle. If \( S_m(N) > S_out(N) \), then the system tends to drift to a higher \( (N) \) state. If \( S_m(N) < S_out(N) \), then the system tends to drift to a lower \( (N) \) state. If \( S_m(N) = S_out(N) \), then the state \( N \) is an equilibrium state. An equilibrium state can be either stable or unstable. If the number of equilibrium states is one, the system is said to be stable; otherwise, it is said to be unstable [12]. If a system has only one stable equilibrium point, say \( N_r \), then the system tends to stay at that state for a long period of time.

Carleial and Hellman [13] introduced the concept of expected drift in their study of the finite population slotted ALOHA model. Jeng [14] proved that the expected drift \( d(N) \) is equal to \( S_m(N) - S_out(N) \). Hence, the two approaches are equivalent.

Suppose now that the system is in state \( N \). Then the mean number of packet flows into the system \( S_m(N) \) is

\[
S_m(N) = (K - N)\sigma,
\]  

(7)

and the mean number of packet flows out of the system, \( S_out(N) \), is

\[
S_out(N) = \left(1 - \frac{\sigma}{\rho}\right) \sum_{i=0}^{N} (1 - P_f(i)) f_t(i)
\]

\[
= N(p - \sigma) \sum_{j=0}^{N-1} (1 - P_f(j + 1))
\]

\[
\cdot \left(\frac{N - 1}{j}\right) p^{j}(1 - p)^{N-1-j}.
\]  

(8)

Fig. 3 shows \( S_m(N) \) and \( S_out(N) \) versus \( N \) for several values of code rate, from which an idea of stabilization can be drawn. It can be noticed that there is a unique equilibrium point (thus stable) at \( N_r = 25 \) if \( r \leq 0.8 \). This suggests that the stability can be ensured by controlling the code rate. That is, given a desired equilibrium point \( N_r \) (which may be obtained from the throughput or delay consideration, i.e., optimum \( N_r \) at which the system performance is maximized), each transmitter controls its code rate \( r = k/n \) such that

\[
\begin{cases} 
S_m(N) \leq S_out(N), & \text{for } N \leq N_r \\
S_m(N) > S_out(N), & \text{for } N > N_r
\end{cases}
\]  

(9)

Then the system will be guaranteed to have a unique equilibrium state at \( N = N_r \) (see Fig. 4). Since \( S_m(N) \) is a strictly decreasing function of \( r \) and \( S_m(N) \) is independent of \( r \), the requirement on code rate (or \( k \)) to have a unique equilibrium state at \( N = N_r \) will be of the form

\[
\begin{cases} 
\rho \geq r_N(N), & N \leq N_r \\
\rho < r_N(N), & N > N_r
\end{cases}
\]  

(10)

where \( r_N(N) \) is the code rate at which \( S_m(N) = S_out(N) \). That is, given a desired equilibrium point \( N_r \), the code rate should be controlled at each \( N \) as in (10) to ensure stability. Then, we cannot only make the network stable, but can also control the equilibrium point as desired. Several \( r_N(N) \)'s are plotted in Fig. 5–6 for two Reed–Solomon codes. These numerical results were obtained from (7), (8), and (9). Inside of the curve is the region for which \( S_m(N) > S_out(N) \) and outside of the curve is the region for which \( S_m(N) < S_out(N) \). We notice that \( r_N(N) \) is insensitive to \( P_0 \) for \( P_0 \leq 10^{-3} \). It can also be noticed that for some regions of small \( N (N \leq 125 \) in Fig. 5 and 6)
network stability cannot be maintained \( r_e(N) = 0 \). This will be clarified in Section V.

Since the value of the code rate \( r \) depends on the state \( N \), it will be necessary for the users to estimate \( N \). Several algorithms for recursive as well as least-square estimation of a random variable have been developed [17], and can be applied here to estimate \( N \). For example, the received signal can be integrated to give an estimate of the total energy in the signals received, which will indicate the number of users currently transmitting and can be used to estimate \( N \).

For Reed–Solomon codes of large block length, \( S_{\text{out}}(N) \) can be obtained from (5), (6), and (8) as

\[
S_{\text{out}}(N) = N(p - \sigma) \sum_{j=0}^{K_m-1} \binom{N-1}{j} p^j(1-p)^{N-1-j}
\]

\[= N(p - \sigma) F \left( \frac{K_m - 1 - (N - 1)p}{\sqrt{(N - 1)p(1-p)}} \right) \tag{11} \]

where we have used the De Moivre–Laplace theorem [15] and

\[ F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\lambda^2/2} d\lambda. \]

The ratio of the two quantities in (11) and (12) tends to 1 as \( N \) tends to infinity. Therefore from (7), (9), and (12), we can get an approximation of \( r_e(N) \), denoted \( \hat{r}_e(N) \), as

\[
\hat{r}_e(N) = (1 - P_0) (1 - P_b) \frac{1}{f(N - 1)p + \sqrt{(N - 1)p(1-p)}} K_m (N\sigma/N(p-\sigma)). \tag{13} \]

The accuracy of this approximation is shown in Fig. 7. We can see that the approximation is very accurate and simple to calculate. We can also notice that \( \hat{r}_e(N) \) is fairly close to \( r_e(N) \) even for \( n = 32 \) in the region of interest, i.e., region of small \( N \).
V. DELAY AND THROUGHPUT

In this section, we find the mean packet delay and the mean channel throughput of the slotted ALOHA frequency-hop packet radio network. These two measures are usually used in the study of the performance analysis of multiple-access protocols, and the former is an especially important parameter for the systems having constraint on maximum delay such as in voice communication systems.

The mean packet delay $D$ in number of time slots can be calculated from the Little’s result [18] as

$$D = \frac{N_r}{\bar{S}(N_r)}$$  \hspace{1cm} (14)

where $\bar{S}(N_r)$ is the mean channel throughput and is given by

$$\bar{S}(N_r) = \sum_{i=0}^{N_r} i (1 - P_E(i)) \binom{N_r}{i} p^i (1 - p)^{N_r - i}.$$  \hspace{1cm} (15)

From (14) and (15) we obtain

$$D = \left[ \sum_{j=0}^{N_r - 1} (1 - P_E(j + 1)) \binom{N_r - 1}{j} p^{j+1} (1 - p)^{N_r - 1 - j} \right]^{-1}.$$  \hspace{1cm} (16)

As the code rate affects the number of information symbols in a packet, we should divide $D$ in (16) by the code rate being used to get the mean delay $D'$ in transmitting a fixed amount of information (to be precise, $D'$ is the number of channel use in transmitting one information symbol). Also, we should multiply $\bar{S}$ in (15) by the code rate and divide by $q$ to get the net information that gets through the network per unit bandwidth. We have plotted $D'$ versus code rate $r$ in Figs. 8 and 9. It can be observed that there exists an optimal code rate that minimizes $D'$. Our empirical data shows that the optimal code rate is about 0.8.

By noting that the maximum of $S_{\text{max}}(N)$ in (8) is $N(p - \sigma)$ and $S_{\text{mix}}(N) = (K - N)\sigma$, we can obtain the
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The mean packet delay \( D \) in number of time slots can be calculated from the Little's result [18] as

\[
D = \frac{N_e}{\overline{S}(N_e)} \tag{14}
\]

where \( \overline{S}(N_e) \) is the mean channel throughput and is given by

\[
\overline{S}(N_e) = \sum_{i=0}^{N_e} i \left( 1 - P_e(i) \right) \binom{N_e}{i} p^i (1-p)^{N_e-i} \tag{15}
\]

From (14) and (15) we obtain

\[
D = \left[ \sum_{j=0}^{N_e - 1} \binom{N_e - 1}{j} \left( 1 - P_e(j+1) \right) \right]^{-1} \cdot p^{i+1} (1-p)^{N_e-1-j} \tag{16}
\]

As the code rate affects the number of information symbols in a packet, we should divide \( D \) in (16) by the code rate being used to get the mean delay \( D' \) in transmitting a fixed amount of information (to be precise, \( D' \) is the number of channel use in transmitting one information symbol). Also, we should multiply \( \overline{S} \) in (15) by the code rate and divide by \( q \) to get the net information that gets through the network per unit bandwidth. We have plotted \( D' \) versus code rate \( r \) in Figs. 8 and 9. It can be observed that there exists an optimal code rate that minimizes \( D' \). Our empirical data shows that the optimal code rate is about 0.8.

By noting that the maximum of \( S_{\text{max}}(N) \) in (8) is \( N(p-a) \) and \( S_{\text{min}}(N) = (K-N)a \), we can obtain the
at $N_e = K \sigma / p$, and the minimum of the mean delay in transmitting a fixed amount of information is attained at around $N_e = K \sigma / p$. The condition on code rate to operate at such a desirable state has also been derived. The maximum mean channel throughput while maintaining the unique equilibrium state at $N_e = K \sigma / p$ has been found.

References


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