6.2
\[ f(x_1) = \frac{n}{i^{n}} \frac{e^{\theta - x_c}}{I(\theta \leq x_c < \infty)} \]
\[ = \frac{n}{i^{n}} \frac{e^{\theta x_c}}{I(\theta \leq \frac{x_c}{i} < \infty)} e^{-x_c} \]
\[ = \frac{n}{i^{n}} \frac{e^{\theta \frac{x_c}{i}}} {I(\theta \leq \min \{ \frac{x_c}{i} \} < \infty)} e^{-x_c} \]

By a factorization theorem \( T = \min \{ x_i/i \} \) is a SS for \( \theta \).

6.3. Let \( x_{(1)} = \min \{ x_i \} \)
\[ f(x | \mu, \sigma) = \frac{n}{i^{n}} \frac{1}{\sigma} \frac{e^{-(x_i - \mu)\sigma}}{I(\mu < x_i < \infty)} \]
\[ = \left( \frac{1}{\sigma} \right)^n \frac{e^{-\sigma x_{(1)} + n\mu}}{I(\mu < x_{(1)} < \infty)} \]

Thus \( (x_{(1)}, \Sigma x_i) \) is a SS for \( \theta \).

6.6.
\[ f(x | \alpha, \beta) = \frac{n}{i^{n}} \frac{1}{P(x)} \frac{\lambda^{-\frac{x_i}{\beta}}}{\Gamma(\alpha)} e^{-\frac{x_i}{\beta}} \]
\[ = \left( \frac{1}{\Gamma(\alpha)} \right) \frac{\alpha^{-\frac{x_{(1)}}{\beta}} e^{-\frac{\Sigma x_i}{\beta}}}{P(x)} \]

Hence \( (\frac{n}{i^{n}} x_i, \Sigma x_i) \) is a SS for \( (\alpha, \beta) \).
Let \( x = (x_1, \ldots, x_n) \) & \( y = (y_1, \ldots, y_n) \)

\[
\frac{f(x|\theta)}{f(y|\theta)} = \frac{\exp(-\frac{1}{2} \sum (x_i - \theta)^2)}{\exp(-\frac{1}{2} \sum (y_i - \theta)^2)} = \exp\left(\frac{1}{2} \left[ \sum x_i^2 - \sum y_i^2 \right] + 2n(\bar{y} - \bar{x}) \right)
\]

which is a constant (free of \( \theta \)) if \( \sum x_i^2 = \sum y_i^2 \) & \( \bar{x} = \bar{y} \).

Hence \( x \), \( y \) is a MSS for \( \theta \).

\[
\frac{f(x|\theta)}{f(y|\theta)} = \frac{n}{\prod_{i=1}^n} e^{-(x_i - \theta)} I(\theta < x_i < \infty) = \frac{\prod_{i=1}^n e^{-(y_i - \theta)} I(\theta < y_i < \infty)}{e^{-\sum y_i} I(\theta < \min y_i < \infty)} \]

which is a constant function of \( \theta \) if \( \min\{x_i\} = \min\{y_i\} \).

Note that \( e^{-\sum x_i}/e^{-\sum y_i} \) is free of \( \theta \) already

and \( \sum x_i \) & \( \sum y_i \) has no role anywhere!

Hence \( T = \min\{x_i\} \) is a MSS.

(c) The whole sample \( (x_1, \ldots, x_n) \) is a MSS as \( \frac{f(x|\theta)}{f(y|\theta)} \) is a constant function of \( \theta \) if \( x = y \).