MATH 373: Homework 4  
“Nonlinear Systems”  
Fall 2013

NOTE: For each homework assignment observe the following guidelines:
• Include a cover page.
• Always clearly label all plots (title, x-label, y-label, and legend).
• Use the subplot command when comparing 2 or more plots to make comparisons easier and to save paper.

1. SOURCE CODE:

Write a function for Newton’s method for systems. Write this function so that it works for a general $m \times m$ system. As part of this code you will have to solve a linear system. Use your LU-decomposition with partial pivoting code from the last homework to solve this linear system. Your function should be written so that it can be called by typing:

$$[\vec{x}, \text{NumIters}] = \text{NewtonSys}(\vec{F}, \vec{J}, \vec{x}_0, \text{TOL}, \text{MaxIters})$$

Here $\vec{F}(\vec{x})$ is the vector-valued function whose root we are trying to approximate and $J(\vec{x})$ is its Jacobian matrix. Include your source code with the rest of your assignment.

2. APPLICATION PROBLEM:

Using your above code, solve the following nonlinear system

$$x_1^3 - 2x_2 - 2 = 0,$$
$$x_1^3 - 5x_3^2 + 7 = 0,$$
$$x_2x_3^2 - 1 = 0.$$ 

3. APPLICATION PROBLEM:

Using your above code, solve the following nonlinear system

$$5 \cos(x) + 6 \cos(x + y) - 10 = 0,$$
$$5 \sin(x) + 6 \sin(x + y) - 4 = 0.$$ 

4. APPLICATION PROBLEM:

Let $i = \sqrt{-1}$. Find all five roots of the complex polynomial

$$f(z) = (1 + i)z^5 - 2z^3 + iz^2 - 1$$
to within a tolerance of $10^{-10}$ using your **Newtons’ method for systems** code. For each root report your initial guess, how many iterations were required for a $10^{-10}$ error, and your approximate root.

**HINT:** The roots of $f(z)$ are the points $z$ in the complex plane where $f(z) = 0$. We can write $z = x + iy$ where $x$ is the real part and $y$ is the imaginary part. In this form, both $x$ and $y$ are strictly real numbers.

We can also separate $f(z)$ into a real part and an imaginary part:

\[
 f(z) = (1 + i)(x + iy)^5 - 2(x + iy)^3 + i(x + iy)^2 - 1
\]

\[
 \Rightarrow f(z) = A(x, y) + iB(x, y).
\]

where

\[
 A(x, y) = x^5 - 5x^4y - 10x^3y^2 + 10x^2y^3 + 5xy^4 - y^5 - 2x^3 + 6xy^2 - 2xy - 1
\]

\[
 B(x, y) = x^5 + 5x^4y - 10x^3y^2 - 10x^2y^3 + 5xy^4 + y^5 - 6x^2y + 2y^3 + x^2 - y^2.
\]

Again, $x$ and $y$ are strictly real numbers and $A(x, y)$ and $B(x, y)$ are strictly real functions.

The roots of $f(z)$ are all the points $z$ such that $f(z) = 0$. Using the above result this is equivalent to saying that the roots of $f(z)$ are all the points $(x, y)$ such that $A(x, y) = B(x, y) = 0$.

5. **APPLICATION PROBLEM:**

The filter coefficients – $h_1, h_2, h_3,$ and $h_4$ – for the Daubechies wavelet of length 4 are solutions of the system

\[
 h_1 + h_2 + h_3 + h_4 = \sqrt{2}
\]

\[
 h_1 - h_2 + h_3 - h_4 = 0
\]

\[
 3h_1 - 2h_2 + h_3 = 0
\]

\[
 h_1^2 + h_2^2 + h_3^2 + h_4^2 = 1.
\]

Determine $h_1, h_2, h_3,$ and $h_4$ using your code from above.