MATH 373: HOMEWORK 3
“LINEAR SYSTEMS”
FALL 2013

NOTE: For each homework assignment observe the following guidelines:

- Include a cover page.
- Always clearly label all plots (title, x-label, y-label, and legend).
- Use the subplot command when comparing 2 or more plots to make comparisons easier and to save paper.

1. Let $B$ be a $4 \times 4$ matrix to which we apply the following operations:
   - double column 1,
   - halve row 3,
   - add row 3 to row 1,
   - interchange columns 1 and 4,
   - subtract row 2 from each of the other rows,
   - replace column 4 by column 3,
   - delete column 1 (so that the column dimension is reduced by 1).

   (a) Write the results as a product of eight matrices.
   (b) Write it again as a product $ABC$ (same $B$) of three matrices.

2. Find the $LU$ factorization of $A$ and use it to solve $Ax = b$ for the following linear system: (Show EACH step in the construction of $L$ and $U$, as well as each step in the forward and backward substitution.)

   \[
   A = \begin{pmatrix}
   4 & 1 & 0 & 0 \\
   1 & 4 & 1 & 0 \\
   0 & 1 & 4 & 1 \\
   0 & 0 & 1 & 4
   \end{pmatrix}, \quad b = \begin{pmatrix}
   3 \\
   -2 \\
   2 \\
   -3
   \end{pmatrix}.
   \]

3. Consider the following linear system

   \[Ax = b,\]

   where $A$ is the following matrix

   \[
   A = \begin{pmatrix}
   0 & 1 & 4 & 5 \\
   2 & 0 & 2 & 4 \\
   2 & 4 & 0 & 1 \\
   1 & -3 & -5 & 0
   \end{pmatrix}.
   \]
(a) Using the partial pivoting strategy, determine the $P, L, U$ decomposition of the matrix $A$, such that $PA = LU$. (Show EACH STEP in the decomposition.)

(b) Use the $P, L, U$ decomposition found in (a) to find the solution to $Ax = \begin{pmatrix} 2 \\ -2 \\ 0 \\ 1 \end{pmatrix}$.

(Show ALL relevant steps).

(c) Use the $P, L, U$ decomposition found in (a) to find the solution to $Ax = \begin{pmatrix} 0 \\ 1 \\ 5 \\ 2 \end{pmatrix}$.

(Show ALL relevant steps).

4. **SOURCE CODE:**

Write the following functions in MATLAB:

- $y = \text{ForwardSubs}(L,b)$ – forward substitution
- $x = \text{BackwardSubs}(U,y)$ – backwards substitution
- $[L,U] = \text{LU}(A)$ – LU decomposition (no pivoting)
- $[P,L,U] = \text{PLU}(A)$ – LU decomposition with partial pivoting

5. Use the above source code to solve Problem 2 and verify that your code and your by-hand calculations agree.

6. Use the above source code to solve Problem 3 and verify that your code and your by-hand calculations agree.