WORKSHEET - T-DISTRIBUTION

1. For each of the following situation, specify the corresponding $t^*$-value. Feel free to sketch the curve of $t$-distribution.
   
   (a) 90th percentile for $df = 24$
   \[ t = 1.318 \]
   
   (b) 2.5th percentile for $df = 12$
   \[ t = -2.179 \]
   
   (c) the $t^*$ values that bound the middle 60% with $df = 5$
   \[ t = -0.92 \quad \text{and} \quad t = 0.92 \]
   
   (d) the 99.75th percentile for $df = 46$
   \[ t = 2.971 \]

2. You want to construct a 98% confidence interval for data based on a sample of size 15. Find the critical value $t^*$ for the construction of the CI.
   \[ t^* = 2.624 \]

3. Find the critical value $t^*$ corresponding to the 95th percentile for each of the following $df$:
   \[ df = 1: 6.314, \quad df = 5: 2.015, \quad df = 15: 1.753, \quad df = 40: 1.684, \quad df = 1000: 1.646 \]
   What would be the value of $z^*$. How does $z^*$ compare to any of the $t^*$ values?
   \[ z^* = 1.645; \ ; z^* \text{ is Smallest} \ ; \text{in fact as } df \text{ increase} \]
   \[ \text{the values of } t^* \text{ decrease and approach } z^* \]

4. In a test $H_0 : \mu = 24$ vs. $H_a : \mu > 24$, the sample data (based on a sample of size 22) yielded a test statistic $t = 2.64$. Find the corresponding p-value. $df=21$:
   \[ 0.005 \leq p\text{-value} < 0.01 \]
   \[ \text{Note: } t = 2.64 \text{ cannot be found in Table D, however we know the value is between 2.518 and 2.831 giving } \]
   \[ \text{no bounds for our p-value} \]

5. In a test $H_0 : \mu = 2$ vs. $H_a : \mu < 2$, the sample data (based on a sample of size 10) yielded a test statistic $t = -1.427$. Find the corresponding p-value. $df=9$:
   \[ \text{look up } t = 1.427 \]
   \[ 0.05 \leq p\text{-value} < 0.10 \]
6. In a test $H_0 : \mu = 15.9$ vs. $H_a : \mu \neq 15.9$, the sample data (based on a sample of size 55) yielded a test statistic $t = 3.31$. Find the corresponding p-value. 

$df = 54$, look up $df = 50$ (always round down)

$2 \times 0.0005 \leq p\text{-value} \leq 2 \times 0.001$

$0.001 \leq p\text{-value} \leq 0.002$

7. In a test $H_0 : \mu = 1.49$ vs. $H_a : \mu > 1.49$, the sample data (based on a sample of size 17) yielded a test statistic $t = 0.54$. Find the corresponding p-value.

$df = 16$

$t = 0.54$ is smaller than the smallest value in Table D for $df = 16$ ($t = .69$)

Because $t = .69$ corresponds to a p-value of 0.25, we know the p-value for $t = 0.54$ must be even larger.

$p\text{-value} > 0.25$

Note: exact p-values can always be found using statistical softwares.