Statistics 601, Fall 2006
Assignment 5

Obtain a copy of the following article:


which may be found using the e-journals link on the library web page.

1. Write the model being used in this paper in an understandable manner. Clearly identify levels of observation and indexing system, the data model, and the random parameter model. Do not worry about prior distributions; discussion of priors starts on page 206 of the paper beginning with the sentence “In the fourth and final . . .”. Also identify parameters to be estimated.

2. For each component in the model you have identified, give a brief (one to several sentence) description of the motivation or justification presented for the specification chosen by the authors.

3. I have always been troubled by inclusion of so-called “overdispersion” terms in models such as the one presented in this paper. Is this entirely reasonable, or is there legitimate cause for concern?

Here, consider a simplified version of the model such as,

\[
Y_{i,j} \sim Po(\lambda_{i,j})
\]

\[
\log(\lambda_{i,j}) = \mu_{i,j} + \epsilon_{i,j}
\]

\[
\mu_{i,j} \sim N(\tau_{i}, \delta^2)
\]

\[
\epsilon_{i,j} \sim N(0, \psi^2_{i})
\]

Compare this model structure with what you would get by not including the
overdispersion terms $\epsilon_{i,j}$ in the model, that is, to the model,

$$Y_{i,j} \sim Po(\lambda_{i,j})$$

$$\log(\lambda_{i,j}) \sim N(\tau_i, \delta^2)$$

See if you can work out forms for the log likelihoods and marginal moments under these models. Compare the moments (at least expected values) to (i) see what the difference is if $\delta$ is the same value in the two models and (ii) whether the models can be made “equivalent” by defining $\delta$ in the simple model to be some function of parameters in the model with overdispersion.

4. The simple model in question 3 looks quite similar to a gamma-Poisson mixture model with varying means as,

$$Y_{i,j} \sim Po(\lambda_{i,j})$$

$$\lambda_{i,j} \sim \text{Gamma} (\alpha, \beta_i)$$

Compare the simple model from question 3 with this gamma-Poisson mixture. Can the models be made equivalent up to the first two marginal moments of the $Y_{i,j}$? If so, are there any remaining differences (hint: you may want to simulate here).

EXTRA

1. If, in the model of question 3 that contains an overdispersion parameter, if one adds an additional level (as is done in the paper) such that,

$$\psi_i \sim \text{Gamma}(\nu_a, \nu_b)$$

does anything change in your assessment from question 3?