Practice problems for Exam I.

The exam will consist of 4 problems (with multiple parts) not unlike these practice problems.

1. A trade union in a certain state would like to determine if the salaries of female bank executives are low as a result of sex discrimination. They obtain a random sample of 50 female executives in the banking business in the state and determine their salaries. From the data they calculate a 95% confidence interval for the population mean.

(a) Identify the population.

(b) Identify the sample.

(c) Identify the variable of interest.

(d) Is inferential or descriptive statistics being applied in this situation?

(e) Use the above example to describe the difference between a population parameter and a sample statistic.
2. A soft drinks company uses a filling machine to fill plastic bottles with cola. The process produces bottles that contain volumes of cola distributed with a mean \( \mu = 300 \text{ ml} \) and standard deviation \( \sigma = 4 \text{ ml} \). The quality and productivity department of the company requires that the bottles contain not less than 290 ml of cola. A random sample of 36 bottles are measured and the sample mean \( \bar{x} \) is calculated from the data.

(a) Compute the probability that a bottle fails to meet the quality requirement.

(b) What is the sampling distribution of \( \bar{x} \)? Justify your answer

(c) Assuming that \( \mu \) is not known, construct a 90% confidence interval for \( \mu \) given that \( \bar{x} = 298 \text{ ml} \).

(d) What would happen to the confidence interval in (c) if the sample size was reduced from 36 to 30?
3. One of the concerns of U. S. industry is the increasing cost of health insurance for its workers. It is believed that the present mean health insurance cost per worker per month is $120. A random sample of 23 small companies (companies with less than $10 million in annual revenues) that offer paid health insurance as a benefit was selected. From the sample mean health insurance cost per worker per month was determined to be $135, and the standard deviation determined to be $32.

(a) Compute a 95% confidence interval for the mean health insurance cost per worker per month. Interpret your answer in terms of the problem.

(b) What assumption must be made for the interval found in (a) to be valid?

(c) Would you conclude that the present mean health insurance cost per worker has changed from the level mentioned in the problem? Justify your answer.

(d) If the sample size increased from 23 to 40, how would the formula for computing the confidence interval in (b) be affected? RECOMPUTATION OF THE INTERVAL IS NOT REQUIRED.
4. It costs more to produce defective items - since they must be scrapped or reworked - than it does to produce nondefective items. This simple fact suggests that manufacturers should ensure the quality of their products by perfecting their production processes rather than through inspection of finished products. In order to better understand a particular metal-shaping process, a manufacturer wishes to estimate the mean length of items produced by the process during the last 24 hours.

(a) How many parts should be sampled in order to estimate the population mean within 0.1 millimeter (mm) with 90% confidence? Previous studies of this machine have indicated that the standard deviation of lengths produced by the stamping operation is about 2mm.

(b) Time permits the use of a sample size no larger than 100. If a 90% confidence interval for \( \mu \) is constructed using \( n=100 \), will it be wider or narrower than would have been obtained using the sample size determined in a)? Explain.
5.
(a) What process parameter is monitored by the $\bar{x}$-chart? By definition, what does it mean if we say that a process is out of control?

(b) Random samples of size $n = 8$ are selected daily for 50 days from a process that produces ball bearings and the diameters measured. The means of the sample means and the ranges of the 50 samples are found to be $\bar{x} = .640$ and $\bar{R} = .015$, respectively. Compute values for the centerline, upper and lower control limits for an $\bar{x}$-chart.

(c) A sample of size $n = 8$ is selected from the process after the control chart was constructed. The sample mean is found to be .647. Is the process still in control? Why or why not?