1. The state of stress at a point in a body with respect to a set of planes parallel to the x, y, and z axes is given by:

\[
\begin{bmatrix}
200 & 40 & -30 \\
40 & 50 & -20 \\
-30 & -20 & -60
\end{bmatrix}
\text{MPa}
\]

(a) Determine the (x,y,z) components of the traction (stress) vector acting on a plane P whose unit normal, \( \mathbf{n} \), is given by \( \mathbf{n} = \frac{3}{\sqrt{19}} \mathbf{e}_x + \frac{3}{\sqrt{19}} \mathbf{e}_y + \frac{1}{\sqrt{19}} \mathbf{e}_z \).

(b) Also, find the normal stress and total shear stress acting on plane P and the direction of the total shear stress on this plane.

(c) Two orthogonal unit vectors that lie in the plane P are given by:

\[
\mathbf{t} = \frac{1}{\sqrt{38}} \mathbf{e}_x + \frac{1}{\sqrt{38}} \mathbf{e}_y - \frac{6}{\sqrt{38}} \mathbf{e}_z
\]

\[
\mathbf{v} = -\frac{1}{\sqrt{2}} \mathbf{e}_x + \frac{1}{\sqrt{2}} \mathbf{e}_y + 0 \mathbf{e}_z
\]

Determine the shear stresses \( \sigma_{nt} \), \( \sigma_{nv} \) acting on the plane P. Show that these two shear stress components combine to produce the total shear stress (in both magnitude and direction) calculated in (b).

(d) Determine the principal stresses and the principal stress directions and the magnitude of the maximum shearing stress

(e) Let the x’, y’, and z’ axes be oriented in the \( \mathbf{n} \), \( \mathbf{t} \), and \( \mathbf{v} \) directions, respectively. Determine the complete state of stress with respect to these (x’,y’,z’) axes. How are your answers here related to your answers in (b) and (c)?
2. The state of strain in a region about a point P in a stressed body is given by

\[
\begin{bmatrix}
\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\
\varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\
\varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz}
\end{bmatrix} = \begin{bmatrix}
0.003 & 0 & -0.003 \\
0 & 0.004 & 0.005 \\
-0.003 & 0.005 & 0.006
\end{bmatrix}
\]

and these strains are constant in this region. Consider the following lines in this region:

(a) Determine the change of the angle (in degrees) between the lines PA and PB, which are initially at right angles to one another in the undeformed body as shown.

(b) Determine the change of the angle (in degrees) between lines PD and PB which are initially at right angles to one another in the undeformed body as shown.

(c) Determine the new length of line PC which originally has a length \(= a\sqrt{2} \) as shown.
(d) Determine the state of strain with respect to the \((x',y',z')\) axes shown. How is this state of strain related to your answers to parts (a) and (b)?

(e) Determine the principal strains and the maximum engineering shear strain at \(P\)

(f) Determine explicitly the direction cosines of the principal strain direction associated with the positive principal strain value, i.e. do not use a software package such as MATLAB to calculate these direction cosines. Show all your work.

(you can use MATLAB or an equivalent package to obtain the answers to the other parts of this problem or to verify your results in this part)

(g) If this state of strain exists in a component made of stainless steel with \(E = 200 \text{ GPa}, \quad \nu = 0.3\), determine the principal stresses and principal stress directions at this point.

(h) Alternatively, if we treat the stainless steel as a transversely isotropic anisotropic solid with stiffness matrix (with respect to the x-y-z axes) given by

\[
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & 1/2 (C_{11} - C_{12}) \\
\end{bmatrix}
\]

where \(C_{11} = 262.7 \text{ GPa}, \quad C_{33} = 216 \text{ GPa}, \quad C_{44} = 129 \text{ GPa}, \quad C_{12} = 98.2 \text{ GPa}, \quad C_{13} = 145 \text{ GPa}\)

determine the principal stresses and principal stress directions
3. An angular plate in a state of plane stress carries stresses of 20 ksi parallel to its sides oriented either as shown in Fig. 3 (a) or as shown in Fig 3 (b). If the stress state is uniform throughout the plate, for both of these cases determine

(a) the stress vector (traction) components $T_x^{(n)}$ and $T_y^{(n)}$ exerted on planes AB and BC.

(b) the stresses $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ in the plate

Figures 3a, 3b.
4. A container is supported by two parallel beams, each with a length of \( L = 40 \text{ ft.} \) and having rectangular cross sections of height \( h = 10 \text{ in.} \) and width \( b = 1 \text{ in.} \). The weight of the container and its contents is \( W = 15,000 \text{ lb.} \) A crack of length \( a \) exists at the bottom of one of the beams at its midpoint as shown in Fig. 4.

(a) if the fracture toughness of the beam material is \( K_c = 120 \text{ ksi}\sqrt{\text{in.}} \), determine the largest crack that could be tolerated before rapid, unstable crack growth would occur.

(b) If an initial crack with \( a = 3.0 \text{ in.} \) exists in the beam, how many times could the container be pushed across the beams before the cracked beam failed? Assume the bending moment varies from zero to a maximum value given by the moment used in part (a) and assume a crack growth law given by the Paris equation

\[
\frac{da}{dN} = A(\Delta K)^m
\]

where \( A = 1 \times 10^{-18} \text{ in./cycle} \) and \( m = 3 \). Also, assume that the configuration factor in the stress intensity expression is equal to a value of 1.12 independent of the crack size.

(c) If an NDE inspection is performed on an average of once every 10,000 crossings, what size of crack must be reliably detected if one wants to guarantee that the beam will not fail before the next inspection? (use the same crack growth law and configuration factor as in part (b))
5. A component carries a biaxial state of stress as shown in Fig. 5a below. These stresses vary in a cyclic manner as shown in Fig. 5b. The material is made of aluminum with a yield stress of 300 MPa and an ultimate stress of 476 MPa (in tension). The S-N curve for this material can be written as

\[ \sigma_a = A \left( N_f \right)^{-B} \]

where \( \sigma_a \) is the alternating stress in MPa, \( N_f \) is the number of cycles to failure, and \( A = 650, B = 0.150 \).

(a) Based on this S-N curve and a Goodman constant life curve, how many cycles can the material undergo before failure? For "effective" mean and alternating stresses use:

\[
\left( \sigma_m \right)_{\text{eff}} = \left( \sigma_{p1}^m + \sigma_{p2}^m + \sigma_{p3}^m \right) / 3
\]

\[
\left( \sigma_a \right)_{\text{eff}} = \sqrt{\left( \sigma_{p1}^a - \sigma_{p2}^a \right)^2 + \left( \sigma_{p1}^a - \sigma_{p3}^a \right)^2 + \left( \sigma_{p2}^a - \sigma_{p3}^a \right)^2 / \sqrt{2}}
\]

(b) At the peak stresses, what is the safety factor with respect to failure by slip, using the von Mises (maximum distortional strain energy) criterion?