Some Answers to Energy Methods Problems:

4.5.3
\[ (u_y)_{\text{Center}} = \frac{5qL^4}{768EI} \]
\[ (\theta)_{\text{Center}} = \frac{7qL^3}{5760EI} \]

4.5.4
(a)
\[ (u_x)_A = \frac{5QL^3}{3EI} \]
\[ (u_y)_A = \frac{QL^3}{EI} \]
\[ (u_z)_A = 0 \]

(b)
\[ (u_x)_A = 0 \]
\[ (u_y)_A = 0 \]
\[ (u_z)_A = \frac{4FL^3}{3EI} + \frac{2FL^3}{GJ} \]

4.6.5
\[ (u_x)_A = \frac{\pi FR^3}{2EI} \]
\[ (u_y)_A = \frac{\pi FR^3}{2EI} \]

\[ \frac{(u_y)_A}{(u_x)_A} = \frac{F_x}{F_y}, \text{ so force and displacement are parallel} \]
\[ k = \frac{F_{\text{total}}}{\Delta_{\text{total}}} = \frac{2EI}{\pi R^3} \]
4.7.7

\[ T = \frac{F}{2 + \frac{6I}{5AL^2}} \]

4.7.12

\[ M_{\text{ends}} = \frac{FL}{8} - \frac{2\beta EI}{L} \]

\[ (u_y)_{\text{center}} = \frac{FL^3}{192EI} + \frac{FL}{4} \quad \text{(down)} \]

4.7.16

(a) axial force and bending moment at F are

\[ N = 0.2453F \]
\[ M = 3.3073FR \]

(b)

\[ (u_y)_{\text{at } F} = 0.0704 \frac{FR^3}{EI} \quad \text{(down)} \]
Finite Element Problems

1. The bar shown (cross sectional area A and Young’s modulus E) is rigidly fixed at its right end and is loaded by a uniform body force $f_0$ (dimensions are force/unit volume) over two thirds of its length.

   ![Bar Diagram](image)

   (a) Using three equal length constant strain finite elements, determine the displacement and stress throughout the bar
   (b) Compare your results of (a) with the exact solution

2. The bar shown has a cross sectional area that varies linearly from $A_0$ at $x = 0$ to $2A_0$ at $x = L$.

   ![Bar Diagram](image)

   (a) Using three equal length finite elements, determine the displacement and stress throughout the bar
   (b) Compare your results of (a) to the exact solution