Introduction to Two-Factor Analysis of Variance

An experiment was conducted to study the effect of floor space and feeder trough space on pig growth. A total of 40 pens of pigs were randomly assigned to four treatments in a completely randomized fashion with 10 pens per treatment. The four treatments considered were

1) low floor space (0.3 m$^2$/pig) and low feeder trough space (2 cm/pig)
2) low floor space (0.3 m$^2$/pig) and high feeder trough space (4 cm/pig)
3) high floor space (0.6 m$^2$/pig) and low feeder trough space (2 cm/pig)
4) high floor space (0.6 m$^2$/pig) and high feeder trough space (4 cm/pig).

The treatments were imposed for 13 weeks beginning when pigs reached 17 days of age. The average weight gained (in kg) by the pigs in each pen was used as the response variable.

1. A factor is another name for an explanatory variable studied in an investigation. Name the two factors in this study.

2. The different values of a factor are called levels. What are the levels of each factor used in this experiment?

3. In a full-factorial experiment, the treatments consist of all possible combinations of levels from the factors being studied. Is this a full-factorial experiment?

4. The treatments are assigned to experimental units. What are the experimental units in this experiment?

5. The usual one-way ANOVA model was fit to the data from the experiment. The residual sum of squares was 555.6. The treatment means (in kg) were as follows.

<table>
<thead>
<tr>
<th>FEEDER TROUGH SPACE</th>
<th>low</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLOOR</td>
<td>56.4</td>
<td>55.8</td>
</tr>
<tr>
<td>SPACE</td>
<td>58.8</td>
<td>61.0</td>
</tr>
</tbody>
</table>

Is there any evidence that the treatments affected weight gain? Conduct one test to answer this question.

6. A general analysis strategy for a two-factor experiment involves first checking model assumptions by examining residuals as in single-factor ANOVA or multiple regression. After transforming the data (if necessary), analysis proceeds with a test for interaction between the two factors. There is interaction between two factors if the effect that one factor has on the response is not the same for all levels of the other factor. Is the effect of the floor-space factor the same for both levels of the feeder-trough-space factor? Conduct one test to answer this question.
7. When there is no significant interaction between factors, it makes sense to average over the levels of the second factor when making comparisons among the levels of the first factor. Conduct one test that compares the average of the low-floor-space treatments to the average of the high-floor-space treatments.

8. Likewise, when there is no significant interaction between factors, it makes sense to average over the levels of the first factor when making comparisons among the levels of the second factor. Conduct one test that compares the average of the low-feeder-trough-space treatments to the average of the high-feeder-trough-space treatments.

9. Suppose the high-floor-space and low-feeder-trough-space mean had been 50.8 kg instead of 58.8 kg. Conduct the test for interaction again using this new data.

10. Explain why the tests conducted in 7 and 8 might no longer be of interest. What tests might be considered instead?
proc glm;
  class floor feeder;
  model y=floor feeder floor*feeder / clparm;
  lsmeans floor feeder floor*feeder;
  estimate 'H floor - L floor' floor 1 -1;
  estimate 'H feeder - L feeder' feeder 1 -1;
run;

The GLM Procedure

Class Level Information
Class     Levels    Values
floor              2    H L
feeder             2    H L
Number of observations  40

Sum of
Source            DF          Squares     Mean Square    F Value    Pr > F
Model              3     170.4000000      56.8000000       3.68    0.0207
Error             36     555.6000000      15.4333333
Corrected Total   39     726.0000000

R-Square    Coeff Var       Root MSE       y Mean
0.234711    6.773324      3.928528      58.00000

Source            DF          Type I SS      Mean Square    F Value    Pr > F
floor             1     144.4000000     144.4000000       9.36    0.0042
feeder            1       6.4000000       6.4000000       0.41    0.5237
floor*feeder      1      19.6000000      19.6000000       1.27    0.2672

Source            DF          Type III SS     Mean Square    F Value    Pr > F
floor             1     144.4000000     144.4000000       9.36    0.0042
feeder            1       6.4000000       6.4000000       0.41    0.5237
floor*feeder      1      19.6000000      19.6000000       1.27    0.2672

Least Squares Means

floor        y LSMEAN
H             59.9000000
L             56.1000000

feeder        y LSMEAN
H             58.4000000
L             57.6000000

floor    feeder        y LSMEAN
H        H             61.0000000
H        L             58.8000000
L        H             55.8000000
L        L             56.4000000

Standard
Parameter           Estimate         Error        t Value    Pr > |t|    95% Confidence Limits
H floor - L floor   3.800000000     1.24230968     3.06        0.0042    1.28047920   6.31952080
H feeder - L feeder  0.800000000     1.24230968     0.64        0.5237    -1.71952080   3.31952080
data two; set one;
  if floor='H' and feeder='L' then y=y-8;
run;

proc glm;
  class floor feeder;
  model y=floor feeder floor*feeder;
  lsmeans floor*feeder / slice=floor;
  lsmeans floor*feeder / slice=feeder;
  estimate 'H floor - L floor when feeder L' floor 1 -1 floor*feeder 0 1 0 -1;
  estimate 'H floor - L floor when feeder H' floor 1 -1 floor*feeder 1 0 -1 0;
  estimate 'H feeder - L feeder when floor L' feeder 1 -1 floor*feeder 0 0 1 -1;
  estimate 'H feeder - L feeder when floor H' floor 1 -1 floor*feeder 1 -1 0 0;
run;

The GLM Procedure

Sum of
Source                      DF         Squares     Mean Square    F  Value    Pr > F
Model                        3      522.400000      174.133333      11.28    <.0001
Error                       36      555.600000       15.433333
Corrected Total             39     1078.000000

R-Square     Coeff Var      Root MSE        y Mean
0.484601      7.015229      3.928528      56.00000

Source                      DF       Type I SS     Mean Square    F Value    Pr > F
floor                        1       0.4000000       0.4000000       0.03    0.8730
feeder                       1     230.4000000     230.4000000      14.93    0.0004
floor*feeder                 1     291.6000000     291.6000000      18.89    0.0001

Source                      DF     Type III SS     Mean Square    F Value    Pr > F
floor                        1       0.4000000       0.4000000       0.03    0.8730
feeder                       1     230.4000000     230.4000000      14.93    0.0004
floor*feeder                 1     291.6000000     291.6000000      18.89    0.0001

Least Squares Means

floor    feeder        y LSMEAN
H        H           61.0000000
H        L           50.8000000
L        H           55.8000000
L        L           56.4000000

floor*feeder Effect Sliced by floor for y

Sum of
floor                      DF         Squares     Mean Square    F  Value    Pr > F
H                           1      520.200000      520.200000      33.71    <.0001
L                           1        1.800000        1.800000       0.12    0.7347

floor*feeder Effect Sliced by feeder for y

Sum of
feeder                      DF         Squares     Mean Square    F  Value    Pr > F
H                            1     135.2000000     135.2000000       8.76    0.0054
L                            1     156.8000000     156.8000000      10.16    0.0030

Parameter                  Estimate  Standard Error   t Value    Pr > |t|
H floor - L floor when feeder L       -5.6000000      1.75689119      -3.19      0.0030
H floor - L floor when feeder H       5.2000000      1.75689119       2.96      0.0054
H feeder - L feeder when floor L       0.6000000      1.75689119      -0.34      0.7347
H feeder - L feeder when floor H       10.2000000      1.75689119       5.81    <.0001