Interpretation of the Slope of the Least-Squares Regression Line

If we regress \( Y \) against \( X \) to get the least-squares regression equation \( \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X \), we can interpret the slope \( \hat{\beta}_1 \) as follows:

- If \( \hat{\beta}_1 > 0 \), we could say something like, “An increase of one unit in \( X \) is associated with an estimated increase of \( \hat{\beta}_1 \) units in the mean of \( Y \).”
- If \( \hat{\beta}_1 < 0 \), we could say something like, “An increase of one unit in \( X \) is associated with an estimated decrease of \( -\hat{\beta}_1 \) units in the mean of \( Y \).”

If we regress \( \log(Y) \) against \( X \) to get the least-squares regression equation \( \hat{\log(Y)} = \hat{\beta}_0 + \hat{\beta}_1 X \), we can interpret the slope \( \hat{\beta}_1 \) as follows:

- “An increase of one unit in \( X \) is associated with an estimated multiplicative change of \( e^{\hat{\beta}_1} \) in the median of \( Y \).”
- Note that if \( \hat{\beta}_1 > 0 \), then the multiplicative factor will be greater than 1, suggesting that the median of \( Y \) increases with increasing \( X \).
- On the other hand if \( \hat{\beta}_1 < 0 \), then the multiplicative factor will be less than 1, suggesting that the median of \( Y \) decreases with increasing \( X \).

If we regress \( Y \) against \( \log(X) \) to get the least-squares regression equation \( \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \log(X) \), we can interpret the slope \( \hat{\beta}_1 \) as follows:

- If \( \hat{\beta}_1 > 0 \), we could say something like, “An increase by a multiplicative factor of 2 in \( X \) is associated with an estimated increase of \( \hat{\beta}_1 \log(2) \) units in the mean of \( Y \).”
- If \( \hat{\beta}_1 < 0 \), we could say something like, “An increase by a multiplicative factor of 2 in \( X \) is associated with an estimated decrease of \( -\hat{\beta}_1 \log(2) \) units in the mean of \( Y \).”

If we regress \( \log(Y) \) against \( \log(X) \) to get the least-squares regression equation \( \hat{\log(Y)} = \hat{\beta}_0 + \hat{\beta}_1 \log(X) \), we can interpret the slope \( \hat{\beta}_1 \) as follows:

- “An increase by a multiplicative factor of 2 in \( X \) is associated with an estimated multiplicative change of \( 2^{\hat{\beta}_1} \) in the median of \( Y \).”
- Note that if \( \hat{\beta}_1 > 0 \), then the multiplicative factor will be greater than 1, suggesting that the median of \( Y \) increases with increasing \( X \).
- On the other hand if \( \hat{\beta}_1 < 0 \), then the multiplicative factor will be less than 1, suggesting that the median of \( Y \) decreases with increasing \( X \).

If a multiplicative factor is between 1 and 2, it is often more clear to describe changes in terms of a percent increase. A multiplicative factor of \( 1.1X \) corresponds to an \( X \% \) increase. For example, a multiplicative factor of 1.42 corresponds to a 42\% increase.

If a multiplicative factor is between 0 and 1, it is often more clear to describe changes in terms of a percent decrease. A multiplicative factor of \( 0.9X \) corresponds to a \( 100 - X \% \) decrease. For example, a multiplicative factor of 0.77 corresponds to a 23\% decrease.