MATH 182 Practice Final

Problem 1. (10 points) Consider the system of difference equations
\[ \begin{align*}
A(t + 1) - A(t) &= B(t) - [A(t)]^2, \\
B(t + 1) - B(t) &= A(t).
\end{align*} \]

a) Rewrite the system in Iterator Form.
b) Given that \( A(0) = 0 \) and \( B(0) = 1 \) evaluate \( A(1), B(1), A(2), B(2) \).

Problem 2. (10 points) Describe the domain of the function of two variables
\[ z = \sqrt{y - 3x^2 + \ln(x)} \]

Problem 3 (10 points)
Evaluate all the second partial derivatives \( f_{xx}, f_{xy}, f_{yx}, f_{yy} \) of the function
\[ z = f(x, y) = \sin(x^2y) \]

Problem 4 (10 points)
Find the minimum of the function
\[ f(x, y) = 2x^2 + y^2 - 2xy - 2x. \]
Apply the second partial test to verify that what you have found is actually a minimum.

Problem 5 (10 points)
Consider the data
\[
\begin{array}{cc}
x & y \\
0 & 1 \\
1 & 2 \\
2 & 1 \\
3 & 1 \\
4 & 2 \\
5 & 0 \\
\end{array}
\]

Use the formulas
\[ m = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} \quad b = \frac{\sum x^2 \sum y - \sum x \sum xy}{n \sum x^2 - (\sum x)^2}, \]
to find the line \( y = mx + b \) the best fits the given data in the least square sense.

Problem 6 (10 points) Evaluate the Average of the function
\[ f(x, y) = x\sqrt{y}, \]
in the rectangular region
\[ 0 \leq x \leq 2, \quad 0 \leq y \leq 3. \]