Five years ago the Iowa State Government changed its hiring procedures from recruitment done by the Iowa Central Personnel Agency (ICPA) to recruitment done by the specific agencies within which hirers would be working. The old procedure for hiring someone into a job involved two steps: First, job seekers would send applications to the ICPA. Second, the ICPA would send the 6 "best applicants" to be interviewed at the specific agency (SA) within which the job was located. In contrast, according to the new procedure all applications are sent to the SA, and officials at the SA choose which are the 6 best applicants to be interviewed.

When the change in hiring procedures was made, the Director of the ICPA justified her decision with the argument that officials at the ICPA were less qualified than those in the SAs to evaluate who the best applicants for a specific job were. Now that her decision has been in effect for 5 years, she has hired you to evaluate whether or not state employees hired since her decision (under the new hiring procedure) are better workers than those hired prior to her decision (under the old hiring procedure). She gives you unlimited access to the ICPA's files on all of Iowa's 3,637,936 past and present state employees. Although most information in these files can NOT be accessed electronically, you are able to compile the following table from the ICPA's main computer:

Table 1. Hiring outcomes of applicants for Iowa state jobs during 2 time periods.

<table>
<thead>
<tr>
<th>Hiring outcome</th>
<th>How long ago a hire was made</th>
<th>6+ years ago</th>
<th>0-5 years ago</th>
</tr>
</thead>
<tbody>
<tr>
<td>hired</td>
<td></td>
<td>3,547,891</td>
<td>90,045</td>
</tr>
<tr>
<td>not hired</td>
<td></td>
<td>14,191,564</td>
<td>359,955</td>
</tr>
</tbody>
</table>

Be sure that you interpret the data in this table correctly. For example, the 3,547,891 in the table indicates the number of applicants who were hired for Iowa state jobs 6 or more years ago.

Your first concern is that one of the two time periods might have had more applicants-per-job than the other. This concerns you because you believe that applicants hired during the last 5 years may be better workers simply because in comparison to previous years more people may have applied for their jobs (or on the other hand, may be worse workers because of fewer applicants). (For example, one would generally expect that the best 6 from among 100 applicants would be worse than the best 6 from among 150 applicants.) Parts a and b of this question deal with this concern.

a. What is the conditional probability that an applicant for an Iowa state job was hired when the old hiring procedure was being used (i.e., given that the hire was made 6+ years ago)?

$$Pr(\text{hired} \mid 6+ \text{ years ago}) = \frac{3,547,891}{17,739,455} = .2$$
b. Using the .05 significance level, evaluate whether during the last 5 years there was a significantly different number of applicants-per-job than during prior years. Was there a substantively different number of applicants-per-job during the last 5 years than during prior years? How can you tell? (Hints: Do NOT use chi-square in your answer. Instead, use the answer in part a as the "no effects value" of the parameter being evaluated. You may find it useful to think here of "employees hired from an applicant pool of a particular size" rather than of applicants-per-job.)

Have both $p_0 = \frac{3,543,891}{17,739,455} = .2$ and $\hat{p} = \frac{90,045}{455,000} = .2001$

$n = 455,000$. Since $2 \times 455,000 = 910,000 > 5$,

use normal approximation to binomial distribution:

$\hat{p} \sim N \left( p, \frac{p(1-p)}{n} \right)$

Moreover, since $2.001 \times 2.001 > 2(1.2)$, use $\chi^2 = \frac{\hat{p}(1-\hat{p})}{n}$

$\chi^2 = \frac{.2001 \times .8006}{455,000} = .8012$

Because $.8012 < \chi^2 < .8012$, there was NOT a significantly different number of applicants-per-job (i.e., the proportion of applicants for which there were jobs was not significantly different from $.2$).

Also, the computed difference (i.e., $|2.001 - .2| = .8001$) is (not only in my opinion but in the opinions of all of my colleagues) not substantively large.

c. If you were to randomly sample 5 applicants who applied for an Iowa state job when the old hiring procedure was being used, what is the probability that all five were hired?

$P(Y=5) = \binom{5}{5}.2^{5}(1-.2)^{5-5} = .2^5 = .032$
Of course, at this point you have not yet considered anything related to whether or not "better workers" have been hired during the past 5 years. In doing this you first identify pairs of employees (comprised of one of the 3,547,891 "pre-decision employees" hired before the ICPA's change in hiring procedures and one of the 90,045 "post-decision employees" hired after the change) who have identical characteristics on a variety of variables (e.g., age, gender, marital status, job-type, etc.). You then randomly sample of 49 of these matched pairs of state employees.

Your data on each pair come from the first-year evaluations written by each employee's supervisor. First, you count the number of positive statements in each of these evaluations. Second, you subtract the number of the positive statements for the pre-decision employee from the number of the positive statements for the post-decision employee in each matched pair. Thus a score of 2 on the resulting variable indicates that the post-decision employee in a matched pair had two more positive statements in her/his first-year evaluation than did the pre-decision employee in the pair. This variable has a mean of 3 and a variance of 100.

d. Do you have statistically significant evidence (at the .05 level) that the ICPA Director's decision resulted in the hiring of better workers? [weight 10]

\[
\bar{x} = 3, \quad \frac{\hat{\sigma}}{\sqrt{n}} = 1.00, \quad \frac{\hat{\sigma}}{\sqrt{49}} = 1.00, \quad n = 49
\]

\[
\frac{\hat{\sigma}}{\sqrt{n}} = \frac{\hat{\sigma}}{\sqrt{49}} = \frac{1.00}{7} = 1.429
\]

Note: \( \mu_0 = 0 \)

No difference (on average) in the number of positive statements.

\[ \Delta = \bar{Z}_{0.05} \cdot 1.429 = 1.645 \cdot 1.429 = 2.35 \]

\[ \Delta = 2.35 \]

Because \( \bar{x} = 3 > 2.35 \), there is statistically significant evidence (at \( \alpha = .05 \)) that the Director's decision resulted in better workers' being hired.
e. In your final report you recommend that someone should replicate (i.e., repeat) your analysis after another 5 years. If such a replication were to find the same sample mean (i.e., 3) and variance (i.e., 100) as you found, what would be the smallest possible sample with which you could conclude (at the .05 significance level) that the ICPA Director's decision resulted in the hiring of better workers?

\[ \Delta = \frac{Z_{\alpha} \cdot \frac{\sigma}{\sqrt{n}}}{3} \]

Thus,

\[ \sqrt{n} = 1.645 \frac{10}{3} \]

and

\[ n = \left(1.645 \frac{10}{3}\right)^2 = 30.067 \]

Round up to 31.