Orthogonal Polynomials

For equally spaced levels of a predictor (explanatory) variable, X, one can code to simplify calculations and interpretations involved with polynomial regression. Given below are formulas for the linear, C_1, and quadratic, C_2, terms as well as a table of the coded values.

$$C_1 = \lambda_1 \left(\frac{X - \overline{X}}{d} \right)$$

where $\overline{X}$ is the mean of X and d is the distance between two consecutive X values.

$$C_2 = \lambda_2 \left(\left(\frac{X - \overline{X}}{d} \right)^2 - \left(\frac{k^2 - 1}{12} \right) \right)$$

where k is the number of levels of X.

	k=3		k=4		k=5		k=6		k=7		k=8		k=9		k=10	
	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2
	–1	1	–3	1	–2	2	–5	5	–3	5	–7	7	–4	28	–9	6
	0	–2	–1	–1	–1	–1	–3	–1	–2	0	–5	1	–3	7	–7	2
	1	1	1	–1	0	–2	–1	–4	–1	–3	–3	–3	–2	– 8	–5	–1
			3	1	1	–1	1	–4	0	–4	–1	–5	–1	–17	–3	–3
					2	2	3	–1	1	–3	1	–5	0	–20	–1	–4
							5	5	2	0	3	–3	1	–17	1	–4
									3	5	5	1	2	– 8	3	–3
											7	7	3	7	5	–1
													4	28	7	2
															9	6
λ	1	3	2	1	1	1	2	$\frac{3}{2}$	1	1	2	1	1	3	2	$\frac{1}{2}$

Example: Temperature, X, has equally spaced levels of 75, 100, and 125 degrees Celsius.

$$C_1 = 1 \left(\frac{X - 100}{25} \right) \qquad C_2 = 3 \left(C_1^2 - \frac{8}{12} \right)$$

X	C_1	C_2
75	-1	1
100	0	–2
125	1	1