

INSTRUCTIONS: You will have 2 hours to complete the exam. The exam has 150 points. Not all questions have the same point value so gauge your time appropriately. Read the questions carefully and completely. Answer each question and show work in the space provided on the exam. Turn in the exam when you are done or when time is up.

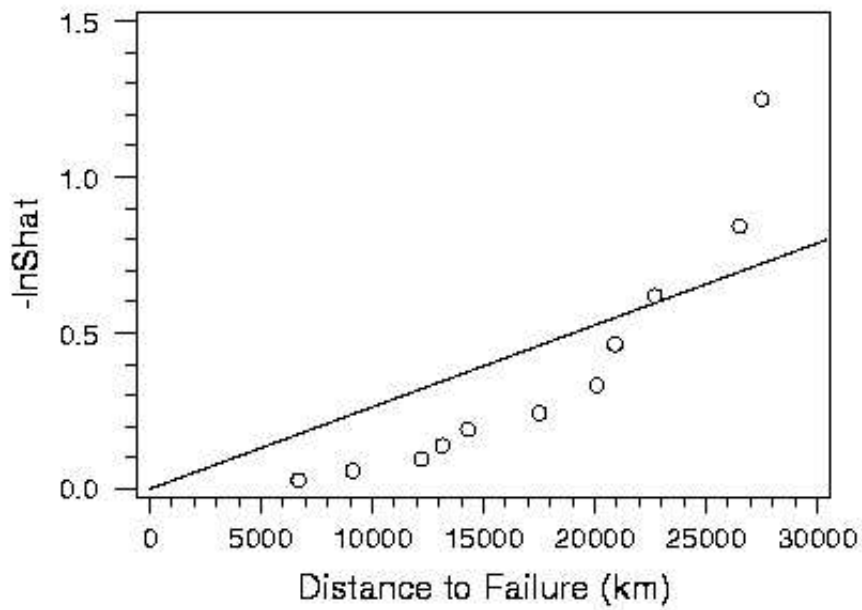
1. [35 pts] In the book *Practical Reliability Engineering*, 2nd edition by P.D.T. O'Connor there are data on the distance to failure for 38 vehicle shock absorbers. For eleven of the shock absorbers there is an actual distance to failure. The remaining 27 are right censored observations, that is the last time the vehicles were checked the shock absorbers had not failed. Below are the data.

| Distance (km) | Failure? | $\hat{S}_t(t)$ | Distance (km) | Failure? | $\hat{S}_t(t)$ |
|---------------|----------|----------------|---------------|----------|----------------|
| 6,700 | Yes | 0.974 | 17,520 | Yes | 0.784 |
| 6,950 | No | | 17,540 | No | |
| 7,820 | No | | 17,890 | No | |
| 8,790 | No | | 18,450 | No | |
| 9,120 | Yes | 0.945 | 18,960 | No | |
| 9,660 | No | | 18,980 | No | |
| 9,820 | No | | 19,410 | No | |
| 11,310 | No | | 20,100 | Yes | |
| 11,690 | No | | 20,100 | No | |
| 11,850 | No | | 20,150 | No | |
| 11,880 | No | | 20,320 | No | |
| 12,140 | No | | 20,900 | Yes | |
| 12,200 | Yes | | 22,700 | Yes | 0.539 |
| 12,870 | No | | 23,490 | No | |
| 13,150 | Yes | | 26,510 | Yes | 0.431 |
| 13,330 | No | | 27,410 | No | |
| 13,470 | No | | 27,490 | Yes | 0.287 |
| 14,040 | No | | 27,890 | No | |
| 14,300 | Yes | | 28,100 | No | |

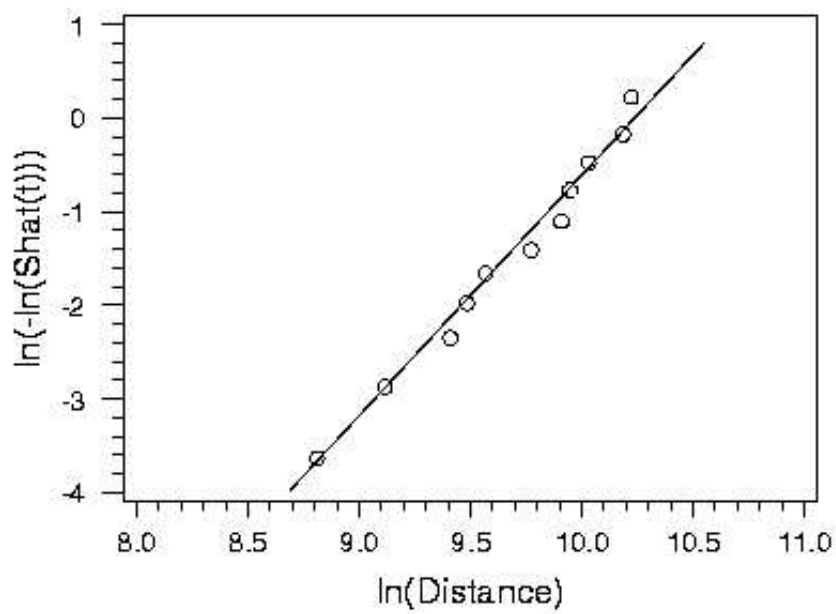
- (a) [5] Fill in the table above by calculating the estimate of the survivor function $\hat{S}_T(t)$ for each of the observed failures.

- (b) [8] What is the estimated probability that a shock absorber will survive longer than 10,000 kilometers? Construct a 95% confidence interval for this probability.
- (c) [4] On the next page are probability plots for these data. According to the plots, which model appears to fit the data better, the Exponential model or the Weibull model?
- (d) [10] Using the model you think is best, estimate the parameter(s) of that model from the probability plot. Give the formula for the survivor function using the estimated parameter(s).
- (e) [4] According to the survivor function in (d), what is the probability that a shock absorber will survive longer than 10,000 kilometers?
- (f) [4] According to the survivor function in (d), what is the median time to failure for the shock absorbers?

Exponential Model
Shock Absorber Distance to Failure



Weibull Model
Shock Absorber Distance to Failure



2. [35] The following problem is adapted from an experiment that was reported in *Rubber Chemistry and Technology*, 1974, pp. 825-836. The effect of silica filler (parts per hundred, pph) and that of oil filler (parts per hundred, pph) on the viscosity of rubber was investigated. In the first experiment a 2^2 experiment with 2 center points was run. The coded values are:

$$X_1 = \frac{(\text{silica} - 60)}{15} \quad X_2 = (\text{oil} - 20)$$

Below are the data.

| silica | oil | viscosity |
|--------|-----|-----------|
| 45 | 19 | 13.71 |
| 75 | 19 | 14.15 |
| 45 | 21 | 12.87 |
| 75 | 21 | 13.53 |
| 60 | 20 | 13.85 |
| 60 | 20 | 13.73 |

- (a) [7] Calculate the estimated full effects of silica, oil and the silica*oil interaction.
- (b) [5] Calculate the variance of the 2 center points and use this to come up with a critical effect size. What effects are statistically significant?
- (c) [5] Is there significant curvature? Give the value of the appropriate test statistic.

A second experiment is conducted to explore further the possible curved relationship between silica, oil and viscosity. Below are the data.

| silica | oil | viscosity |
|--------|------|-----------|
| 38.8 | 20 | 12.99 |
| 81.2 | 20 | 13.89 |
| 60 | 18.6 | 14.16 |
| 60 | 21.4 | 12.90 |
| 60 | 20 | 13.84 |
| 60 | 20 | 13.76 |

- (d) [7] Is there a statistically significant difference between the average at the center points for the first experiment and the average at the center points for the second experiment? Give the value of the appropriate test statistic. Hint: We have not done this test in class, but you should be able to adapt the test for curvature to answer this question. Alternatively, you can think of this as a single factor with two levels: Experiment 1 and Experiment 2.
- (e) [3] A model that included linear and quadratic effects was fit to the combined data from the two experiments. The output from Minitab appears on the next page. Give the prediction equation.
- (f) [4] According to the model in (e), what is the predicted viscosity when silica=67.5 and oil=19.5?
- (g) [4] On the next page is the contour plot for the model given in (e). If we wish to have a viscosity of 14 while using the least amount of oil, what level of silica should we use?

Response Surface Regression

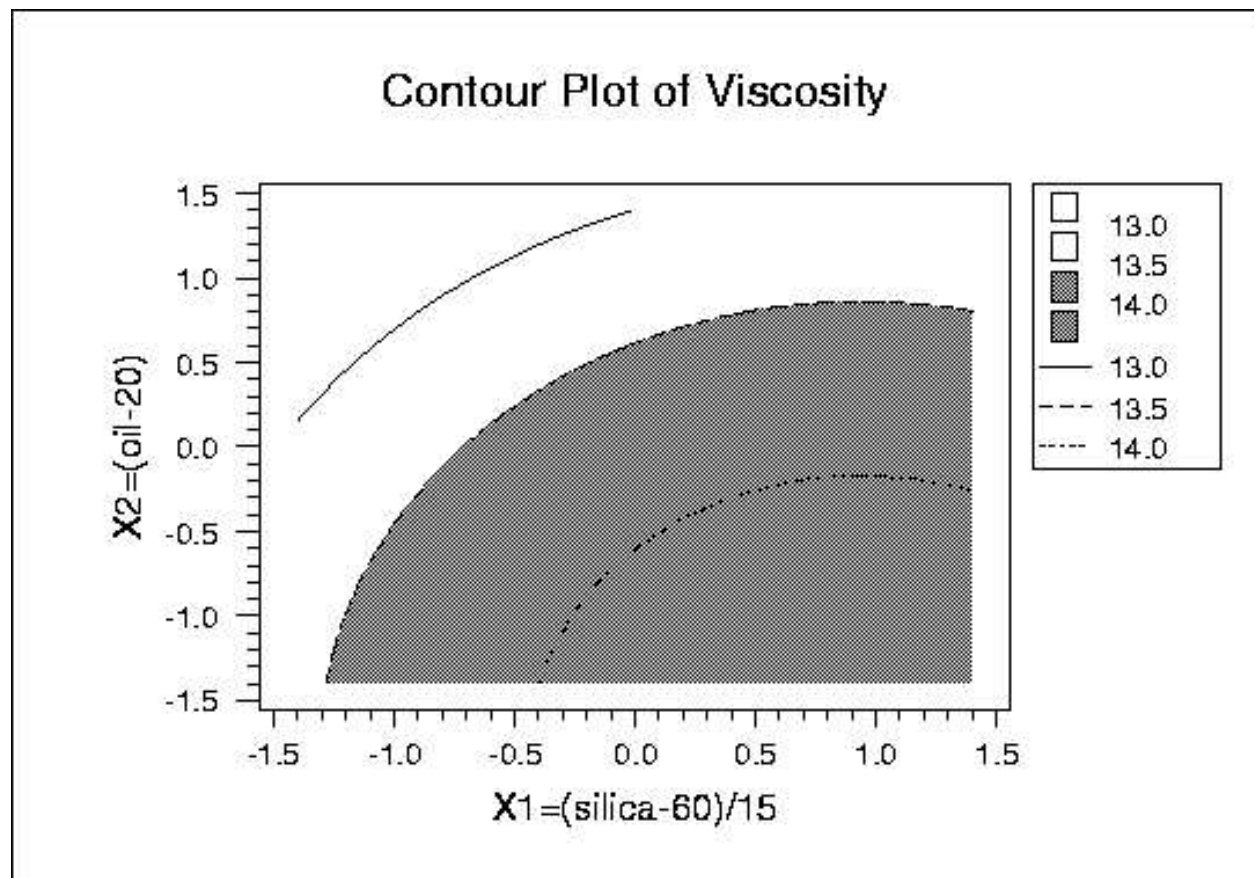
Estimated Regression Coefficients for viscos

| Term | Coef | Stdev | t-ratio | p |
|----------|---------|---------|---------|-------|
| Constant | 13.7941 | 0.04463 | 309.107 | 0.000 |
| X1 | 0.2980 | 0.03172 | 9.395 | 0.000 |
| X3 | -0.4071 | 0.03172 | -13.834 | 0.000 |
| X1*X1 | -0.1587 | 0.03578 | -4.434 | 0.003 |
| X3*X3 | -0.1138 | 0.03578 | -3.151 | 0.016 |

s = 0.08926 R-sq = 97.5% R-sq(adj) = 96.1%

Analysis of Variance for viscos

| Source | DF | Seq SS | Adj SS | Adj MS | F | P |
|----------------|----|---------|---------|---------|--------|-------|
| Regression | 4 | 3.31633 | 3.31633 | 0.55408 | 69.54 | 0.000 |
| Linear | 3 | 3.01563 | 3.01563 | 1.00781 | 126.49 | 0.000 |
| Square | 1 | 0.30070 | 0.30070 | 0.10035 | 13.59 | 0.005 |
| Residual Error | 7 | 0.05577 | 0.05577 | 0.00797 | | |
| Lack-of-Fit | 4 | 0.04527 | 0.04527 | 0.01132 | 3.33 | 0.181 |
| Pure Error | 3 | 0.01050 | 0.01050 | 0.00350 | | |
| Total | 11 | 3.37210 | | | | |

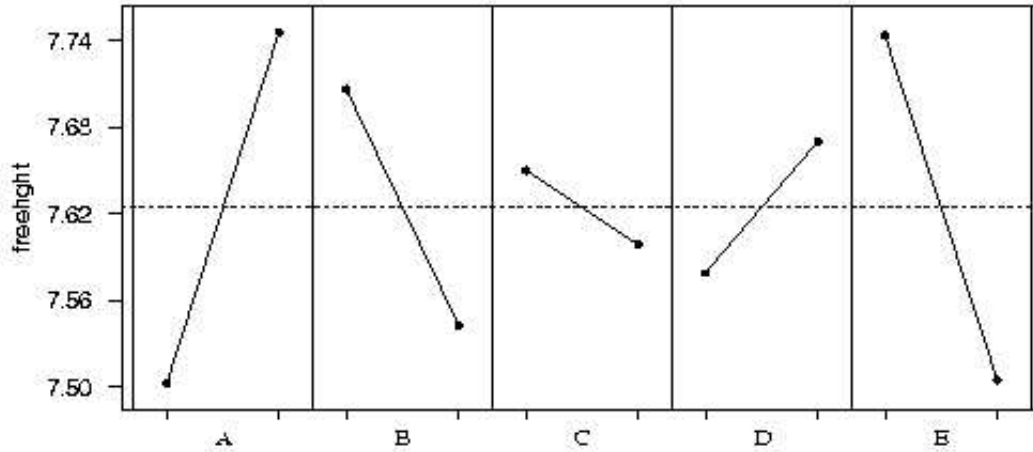


3. [45 pts] An article in the *Journal of Quality Technology* (1985), Vol. 17, pp. 198-206, describes a replicated fractional factorial. The experiment looked at the effect of 5 factors on the free height of leaf springs used in an automotive application. The factors are: A=furnace temperature, B=heating time, C=transfer time, D=hold down time, and E=quench oil. The data are given below.

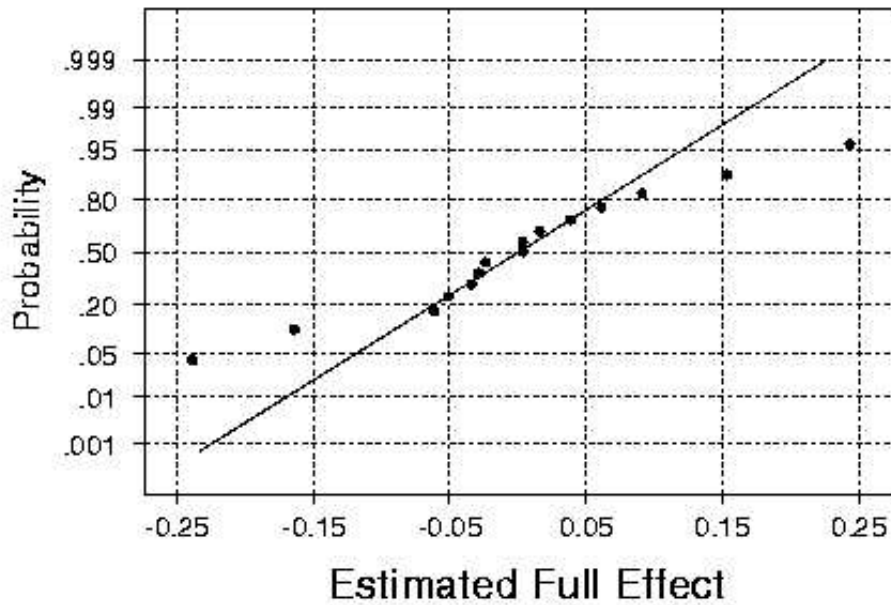
| A | B | C | D | E | free height | | | \bar{X} | s^2 |
|---|---|---|---|---|-------------|------|------|-----------|--------|
| - | - | - | - | - | 7.78 | 7.78 | 7.81 | 7.79 | 0.0003 |
| + | - | - | + | - | 8.15 | 8.18 | 7.88 | 8.07 | 0.0273 |
| - | + | - | + | - | 7.50 | 7.56 | 7.50 | 7.52 | 0.0012 |
| + | + | - | - | - | 7.59 | 7.56 | 7.73 | 7.63 | 0.0093 |
| - | - | + | + | - | 7.54 | 7.98 | 7.88 | 7.80 | 0.0532 |
| + | - | + | - | - | 7.70 | 8.09 | 8.06 | 7.95 | 0.0471 |
| - | + | + | - | - | 7.56 | 7.52 | 7.42 | 7.50 | 0.0052 |
| + | + | + | + | - | 7.56 | 7.81 | 7.70 | 7.60 | 0.0157 |
| - | - | - | - | + | 7.50 | 7.25 | 7.12 | 7.29 | 0.0373 |
| + | - | - | + | + | 7.88 | 7.87 | 7.44 | 7.73 | 0.0631 |
| - | + | - | + | + | 7.50 | 7.56 | 7.50 | 7.52 | 0.0012 |
| + | + | - | - | + | 7.63 | 7.75 | 7.57 | 7.65 | 0.0084 |
| - | - | + | + | + | 7.37 | 7.44 | 7.44 | 7.40 | 0.0048 |
| + | - | + | - | + | 7.55 | 7.69 | 7.62 | 7.62 | 0.0049 |
| - | + | + | - | + | 7.17 | 7.18 | 7.25 | 7.20 | 0.0019 |
| + | + | + | + | + | 7.80 | 7.50 | 7.59 | 7.63 | 0.0237 |

- (a) [3] What fraction of a full factorial is this?
- (b) [3] Write down the defining relationship for this experiment. Hint: Look at the +/-'s for the factor D, with what interaction is this confounded?
- (c) [3] What is the resolution?
- (d) [5] Is this the highest resolution one can obtain given the number of factors and number of runs? If so, explain why? If not, explain how one could have obtained a higher resolution.

Main Effects Plot for Free Height



Normal Probability Plot



Fractional Factorial Fit

Estimated Effects and Coefficients for freehght

| Term | Effect | Coef | Std Coef | t-value | P |
|----------|---------|---------|----------|---------|-------|
| Constant | | 7.6344 | 0.01993 | 383.84 | 0.000 |
| A | 0.3438 | 0.1319 | 0.01993 | 6.13 | 0.000 |
| B | -0.1638 | -0.0819 | 0.01993 | -4.11 | 0.000 |
| C | -0.0513 | -0.0356 | 0.01993 | -1.39 | 0.307 |
| D | 0.0912 | 0.0456 | 0.01993 | 2.29 | 0.029 |
| E | -0.3387 | -0.1194 | 0.01993 | -5.99 | 0.000 |
| A*B | -0.0387 | -0.0144 | 0.01993 | -0.73 | 0.476 |
| A*C | 0.0038 | 0.0019 | 0.01993 | 0.09 | 0.936 |
| A*D | -0.0338 | -0.0119 | 0.01993 | -0.60 | 0.555 |
| A*E | 0.0613 | 0.0306 | 0.01993 | 1.54 | 0.134 |
| B*E | 0.1538 | 0.0769 | 0.01993 | 3.86 | 0.001 |
| C*E | -0.0337 | -0.0169 | 0.01993 | -0.85 | 0.403 |
| D*E | 0.0388 | 0.0194 | 0.01993 | 0.97 | 0.338 |
| A*B*E | 0.0038 | 0.0019 | 0.01993 | 0.09 | 0.936 |
| A*C*E | 0.0163 | 0.0081 | 0.01993 | 0.41 | 0.686 |
| A*D*E | -0.0613 | -0.0306 | 0.01993 | -1.54 | 0.134 |

Analysis of Variance for freehght

| Source | DF | Seq SS | Adj SS | Adj MS | F | P |
|--------------------|----|---------|---------|---------|-------|-------|
| Main Effects | 5 | 1.85019 | 1.85019 | 0.37004 | 19.44 | 0.000 |
| 2-Way Interactions | 7 | 0.37723 | 0.37723 | 0.05389 | 2.83 | 0.021 |
| 3-Way Interactions | 3 | 0.04836 | 0.04836 | 0.01612 | 0.85 | 0.479 |
| Residual Error | 32 | 0.60920 | 0.60920 | 0.01904 | | |
| Pure Error | 32 | 0.60920 | 0.60920 | 0.01904 | | |
| Total | 47 | 2.88498 | | | | |

- (e) [5] Looking at the main effects plot describe the size and direction of the five main effects on the free height of springs.
- (f) [5] On the normal probability plot, label the points, with name and alias, that appear to be significant.
- (g) [8] What is the standard error of an estimated full effect? What effects, give names and aliases, are statistically significant?
- (h) [4] Given the confounding, how do you attribute the effects found to be significant above? Briefly explain your reasoning.
- (i) [3] Give the prediction equation, include only those effects found to be significant, for free height.
- (j) [6] If we wish to maximize free height, what should the settings for the factors be? What is the predicted maximum free height value for those settings?

4. [20 pts] You are planning an experiment with four factors, each at two levels. You are limited to 16 runs. The runs will be completely randomized. The three possible experiments you could run are: (1) A single replicate of a full 2^4 factorial, (2) Two replicates of a $\frac{1}{2}$ fraction of a 2^4 , a 2_{IV}^{4-1} , or (3) A $\frac{1}{2}$ fraction of a 2^4 , a 2_{IV}^{4-1} , with 8 center points.
- (a) [6] For each experiment indicate what effects you can estimate uniquely or how effects will be confounded.
- (b) [9] For each experiment indicate how you would estimate error variability and how many degrees of freedom you would have for that estimate.
- (c) [5] Which experiment would you run? Briefly justify your choice.

5. [15 pts] Taguchi connects variability with loss/cost. Briefly describe this connection. Does this viewpoint agree or disagree with the notion of variation presented in this course, Applied Statistics for Industry? Explain briefly.

Notice for students at remote sites.

Because most of you will take the final exam and turn in your project after grades are due at the Registrar's Office, you may receive notification from the Registrar's Office that either no grade has been submitted or an Incomplete grade has been submitted. Please ignore this if it is sent to you. I will return graded final exams, critiques of the project and course grades by the week of June 2, 1997.