

Formulas for Exam 1

• Process Capability

Theoretical	Estimate
$C_p = \frac{USL-LSL}{6\sigma}$	$\hat{C}_p = \frac{USL-LSL}{6s}$
$CPU = \frac{USL-\mu}{3\sigma}$	$C\hat{P}U = \frac{USL-\bar{X}}{3s}$
$\% > USL = Pr(Z > 3CPU)$	$\% > USL = Pr(Z > 3C\hat{P}U)$
$CPL = \frac{\mu-LSL}{3\sigma}$	$C\hat{P}L = \frac{\bar{X}-LSL}{3s}$
$\% < LSL = Pr(Z > -3CPL)$	$\% < LSL = Pr(Z > -3C\hat{P}L)$
$C_{pk} = \min(CPU, CPL)$	$\hat{C}_{pk} = \min(C\hat{P}U, C\hat{P}L)$
$m = \frac{LSL+USL}{2}$	$m = \frac{LSL+USL}{2}$
$k = \frac{2 m-\mu }{(USL-LSL)}$	$\hat{k} = \frac{2 m-\bar{X} }{(USL-LSL)}$
$C_{pk} = C_p(1 - k)$	$\hat{C}_{pk} = \hat{C}_p(1 - \hat{k})$

$$s = \sqrt{\frac{\sum(X_i - \bar{X})^2}{N - 1}}$$

One can also use: $\hat{\sigma} = \frac{\bar{R}}{d_2}$ or $\tilde{\sigma} = \frac{\bar{s}}{c_4}$ instead of s as an estimate of σ in any of the formulas.

- **Analysis of Variance: k groups, n obs. per group**

$$SS_{Within} = SS_{repError} = \sum(n-1)s_i^2$$

$$df = \sum(n-1) = k(n-1)$$

$$MS_{Within} = MS_{repError} = s_p^2 = \frac{SS_{Within}}{k(n-1)}$$

$$LSD = t_{\alpha/2, k(n-1)} \sqrt{s_p^2} \sqrt{\frac{2}{n}}$$

Replace $t_{\alpha/2, k(n-1)}$ by the value 2. If you want greater confidence use the value of 3 instead.

- **Least Squares Regression**

$$\hat{m}_1 = \frac{(\sum C_{1i} \bar{Y}_i)}{(\sum C_{1i}^2)} \quad \hat{b} = \bar{Y} \quad \hat{Y} = \hat{b} + \hat{m}_1 C_1$$

$$SS_{Linear} = (\hat{m}_1)^2 n (\sum C_{1i}^2) \quad df_{Linear} = 1$$

$$t_{Linear} = \sqrt{F} = \sqrt{\frac{MS_{Linear}}{MS_{repError}}}$$

The linear term is significant if t_{Linear} is greater than 2 or 3.

$$\hat{m}_2 = \frac{(\sum C_{2i} \bar{Y}_i)}{(\sum C_{2i}^2)} \quad \hat{Y} = \hat{b} + \hat{m}_1 C_1 + \hat{m}_2 C_2$$

$$SS_{Quad} = (\hat{m}_2)^2 n (\sum C_{2i}^2) \quad df_{Quad} = 1$$

$$t_{Quad} = \sqrt{F} = \sqrt{\frac{MS_{Quad}}{MS_{repError}}}$$

The quadratic term is significant if t_{Quad} is greater than 2 or 3.

$$R^2 = \frac{SS_{Model}}{SS_{Total}}$$