

Chemical Product: Nested Design

| Batch | Sample | Measured Purity (%) | | mean | variance |
|-------|--------|---------------------|-----------------|-------|----------|
| | | Determination 1 | Determination 2 | | |
| 1 | 1 | 98.4 | 99.3 | 98.85 | 0.405 |
| | 2 | 98.6 | 98.2 | 98.40 | 0.080 |
| 2 | 1 | 97.6 | 98.1 | 97.85 | 0.125 |
| | 2 | 97.4 | 97.4 | 97.40 | 0.000 |
| 3 | 1 | 97.9 | 98.9 | 98.40 | 0.500 |
| | 2 | 98.4 | 97.9 | 98.15 | 0.125 |
| 4 | 1 | 98.7 | 98.3 | 98.50 | 0.080 |
| | 2 | 99.6 | 99.3 | 99.45 | 0.045 |
| 5 | 1 | 95.7 | 96.0 | 95.85 | 0.045 |
| | 2 | 94.6 | 94.9 | 94.75 | 0.045 |
| 6 | 1 | 95.0 | 95.3 | 95.15 | 0.045 |
| | 2 | 95.1 | 94.9 | 95.00 | 0.020 |
| 7 | 1 | 96.9 | 96.3 | 96.60 | 0.180 |
| | 2 | 96.7 | 96.7 | 96.70 | 0.000 |
| | | | | | 1.695 |

$$SS_{Determination} = (2 - 1) \cdot (1.695) = 1.695$$

| Batch | mean | |
|-------|--------|-----------------------------------|
| 1 | 98.625 | |
| 2 | 97.625 | |
| 3 | 98.275 | variance of the Batch means is |
| 4 | 98.975 | |
| 5 | 95.300 | 2.4906 |
| 6 | 95.075 | |
| 7 | 96.650 | |

$$SS_{Batch} = 2 \cdot 2 \cdot (7 - 1) \cdot (2.4906) = 59.774$$

| Batch | Sample | mean | variance |
|-------|--------|-------|----------|
| 1 | 1 | 98.85 | |
| | 2 | 98.40 | 0.10125 |
| 2 | 1 | 97.85 | |
| | 2 | 97.40 | 0.10125 |
| 3 | 1 | 98.40 | |
| | 2 | 98.15 | 0.03125 |
| 4 | 1 | 98.50 | |
| | 2 | 99.45 | 0.45125 |
| 5 | 1 | 95.85 | |
| | 2 | 94.75 | 0.60500 |
| 6 | 1 | 95.15 | |
| | 2 | 95.00 | 0.01125 |
| 7 | 1 | 96.60 | |
| | 2 | 96.70 | 0.00500 |
| | | | 1.30625 |

$$SS_{Sample} = 2 \cdot (2 - 1) \cdot (1.30625) = 2.613$$

| Source | df | SS | Mean Square | Expected Mean Square |
|---------------|----|--------|-------------|---|
| Batch | 6 | 59.774 | 9.962 | $\sigma_{Det}^2 + 2 \cdot \sigma_{Samp}^2 + 4 \cdot \sigma_{Batch}^2$ |
| Sample | 7 | 2.613 | 0.373 | $\sigma_{Det}^2 + 2 \cdot \sigma_{Samp}^2$ |
| Determination | 14 | 1.695 | 0.121 | σ_{Det}^2 |
| Total | 27 | 64.082 | | |