Data from Batch Chemical Process[1]

Table 1: Raw Data

	Batch 1		Batch 2		Batch 3		Batch 4	
Sample	Test 1	Test 2	Test 1	Test 2	Test 1	Test 2	Test 1	Test 2
1	15.5	15.0	15.4	14.8	12.9	12.3	11.6	11.3
2	15.8	15.2	13.9	13.3	13.5	13.8	12.1	12.5
3	14.0	14.5	13.4	14.7	14.9	15.7	13.1	13.5

Table 2: Cell Means and Variances

	Batch 1		Batch 2		Batch 3		Batch 4	
Sample	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
1	15.25	0.125	15.10	0.180	12.60	0.180	11.45	0.045
2	15.50	0.180	13.60	0.180	13.65	0.045	12.30	0.080
3	14.25	0.125	14.05	0.845	15.30	0.320	13.30	0.080

Table 3: Batch Means and Variances of Cell Means

Batch 1		Batch 2		Batch 3		Batch 4	
Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
15.00	0.4375	14.25	0.5925	13.85	1.8525	12.35	0.8575

[1]From “Nested Designs: A Tool for Process Engineers” by Peter B. Peterka and W. Robert Stephenson, *Chemical Engineering Progress*, 1990, Vol. 86, No. 4, p 12-15.

Formulas for Nested Designs

Sums of Squares

SS_{Total} = (abc-1)(grand sample variance of all **abc** data points)

$SS_{C(B(A))}$ = (c-1)(sum of the **ab** sample variances of the C(B(A)) units)

$SS_{B(A)}$ = c(b-1)(sum of the **a** sample variances of the B(A) means)

SS_A = cb(a-1)(the sample variance of the A means)

Analysis of Variance

Source	Sum of Squares	d.f.	Mean Square	Expected Mean Square
A	SS_A	a-1	SS_A/(a-1)	$\sigma^2 + c\sigma_B^2 + bc\sigma_A^2$
B(A)	$SS_{B(A)}$	a(b-1)	$SS_{B(A)}$/a(b-1)	$\sigma^2 + c\sigma_B^2$
C(B(A))	$SS_{C(B(A))}$	ab(c-1)	$SS_{C(B(A))}$/ab(c-1)	σ^2
Total	SS_{Total}	abc-1		

Variance Components

$$\hat{\sigma}^2 = MS_{C(B(A))}$$

$$\hat{\sigma}_B^2 = \frac{1}{c}[MS_{B(A)} - MS_{C(B(A))}]$$

$$\hat{\sigma}_A^2 = \frac{1}{bc}[MS_A - MS_{B(A))}]$$