

Control Charts for Attributes

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1 Charts for Attributes: Extra Variation

One of the assumptions behind the c- and U-charts is that the occurrence of a defect is independent of the occurrence of other defects. However, often times defects occur in clusters. Clustering of defects produces more variation than is accounted for in the c- and U-chart control limits. For this reason, some practitioners recommend using g- and h-charts to account for this extra variation due to clustering.

As integrated circuits (ICs) continue to grow in area and density, the issue of defect clusters becomes more important. Currently, IC board manufacturers assume a Poisson distribution for nonconformities or defects. This approach gives accurate results for assemblies that are not very complex. However, for complex boards the Poisson model tends to underestimate the variability in the number of nonconformities due to clustering. In particular, solder defects tend to be clustered. The table below gives data on the number of solder defects for 50 boards.

Number of solder defects for 50 circuit boards

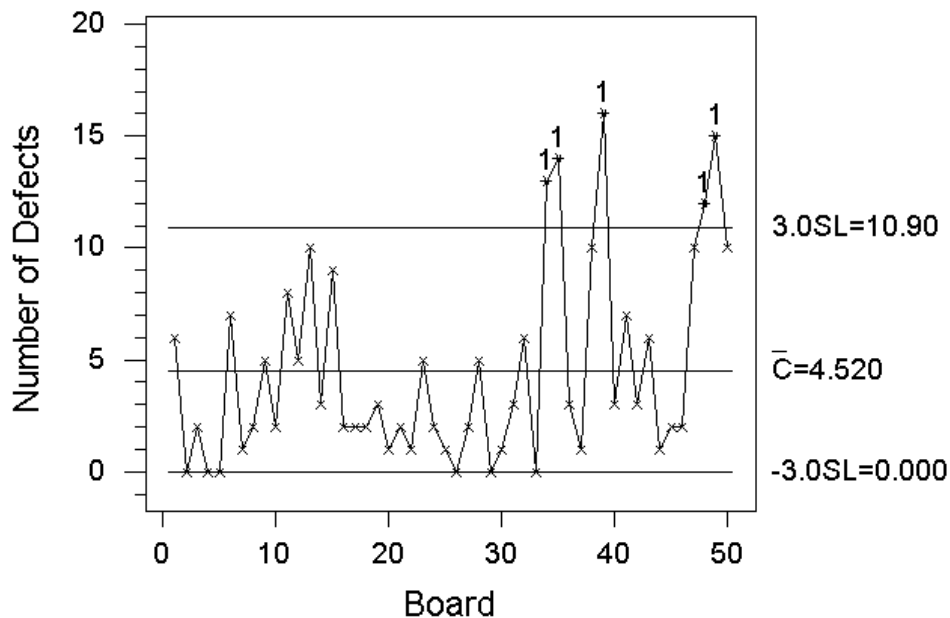
| Board | Defects | Board | Defects | Board | Defects | Board | Defects |
|-------|---------|-------|---------|-------|---------|-------|---------|
| 1 | 6 | 16 | 2 | 31 | 3 | 46 | 2 |
| 2 | 0 | 17 | 2 | 32 | 6 | 47 | 10 |
| 3 | 2 | 18 | 2 | 33 | 0 | 48 | 12 |
| 4 | 0 | 19 | 3 | 34 | 13 | 49 | 15 |
| 5 | 0 | 20 | 1 | 35 | 14 | 50 | 10 |
| 6 | 7 | 21 | 2 | 36 | 3 | | |
| 7 | 1 | 22 | 1 | 37 | 1 | | |
| 8 | 2 | 23 | 5 | 38 | 10 | | |
| 9 | 5 | 24 | 2 | 39 | 16 | | |
| 10 | 2 | 25 | 1 | 40 | 3 | | |
| 11 | 8 | 26 | 0 | 41 | 7 | | |
| 12 | 5 | 27 | 2 | 42 | 3 | | |
| 13 | 10 | 28 | 5 | 43 | 6 | | |
| 14 | 3 | 29 | 0 | 44 | 1 | | |
| 15 | 9 | 30 | 1 | 45 | 2 | | |

The average number of defects for the 50 boards is $\bar{c} = 4.52$. A c-chart for the number of defects would have 4.52 as its center line and control limits at:

$$4.52 \pm 3\sqrt{(4.52)}$$

That control chart appears below. There are several signals of boards with numbers of defects above the upper control limit. These correspond to boards 34, 35, 39, 48 and 49. Using this chart, the local work force would be in charge of searching for and eliminating the special cause that produces these unusually high values.

c-Chart of Solder Defects on Circuit Boards



Alternatively, if we believe that clustering is occurring then the variation used to set the control limits in the c-chart is understated. If clustering is an ongoing problem that affects the process all of the time, then it is part of the system of common cause variation. With this in mind, the values for the control limits can be calculated using:

$$\bar{c} \pm 3\sqrt{\bar{c}(\bar{c} + 1)}$$

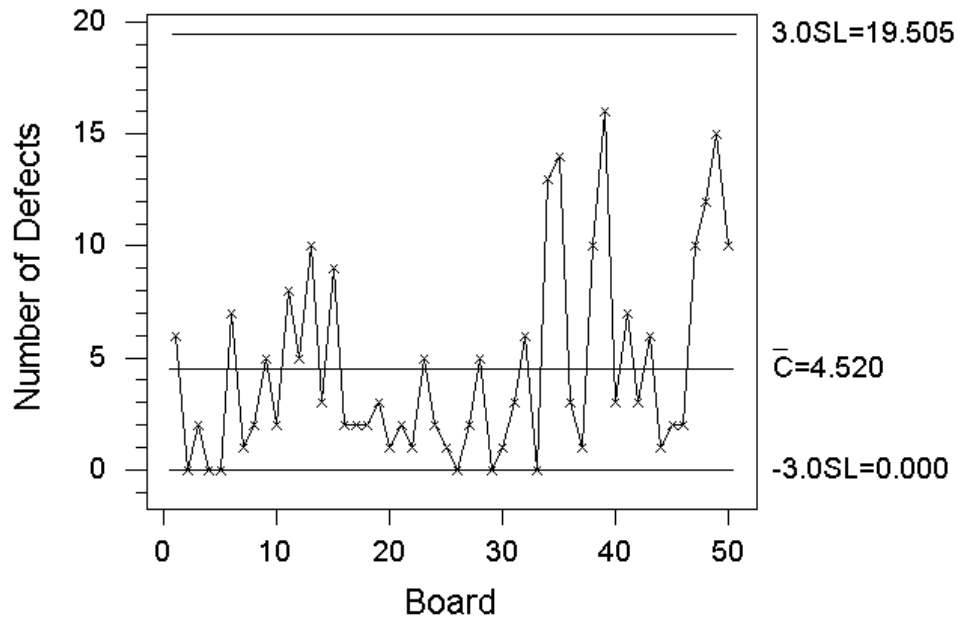
In our example this would give upper and lower control limits of:

$$4.52 \pm 3\sqrt{4.52(5.52)} = 4.52 \pm 14.985$$

Since the calculated value of the lower control limit is less than zero, we set it equal to zero.

The new control chart, the g-chart is given below. Now there are no points plotting outside of the control limits. However, an upper bound of practically 20 defects per board is not very desirable. At this point the system should be looked at to see how numbers of defects can be reduced. One obvious way to improve the system would be to change it so that clusters of defects do not occur.

g-Chart of Solder Defects on Circuit Boards



The two charts are quite different in terms of sounding alarms. The c-chart sounds many alarms while the g-chart sounds none. Some would argue that the c-chart is doing a better job. However, if we remember the distinction between special cause and common cause and the responsibility assignments of Deming, these differences are put in perspective. If we really believe that the clusters are due to some intermittent phenomena that can be corrected by the local workforce then the c-chart gives the appropriate picture and clearly says clustering is a special cause that is not part of the system. On the other hand, if clustering is a system related problem then the c-chart and the implied responsibility for the local workforce to fix the problem can lead to great frustration on the part of that workforce. The g-chart indicates that with the present system, the process will continue to produce solder defects, sometimes as many as 20 per board. If this is unacceptable then management needs to address the problem by changing the system.

Both charts display the data, how we view that data and whose responsibility it is to improve the process differs depending on which chart is used.