Formulas for Final Exam

- **Process Capability**

Theoretical	Estimate
$C_p = \frac{USL-LSL}{6\sigma}$	$\hat{C}_p = \frac{USL-LSL}{6s}$
$CPU = \frac{USL-\mu}{3\sigma}$	$C\hat{P}U = \frac{USL-\overline{\overline{X}}}{3s}$
$CPL = \frac{\mu-LSL}{3\sigma}$	$C\hat{P}L = \frac{\overline{\overline{X}}-LSL}{3s}$
$C_{pk} = min(CPU, CPL)$	$\hat{C}_{pk} = min(C\hat{P}U, C\hat{P}L)$
$m = \frac{LSL+USL}{2}$	$m = \frac{LSL+USL}{2}$
$k = \frac{2\|m-\mu\|}{(USL-LSL)}$	$\hat{k} = \frac{2\|m-\overline{\overline{X}}\|}{(USL-LSL)}$
$C_{pk} = C_p(1-k)$	$\hat{C}_{pk} = \hat{C}_p(1-\hat{k})$

$$s = \sqrt{\frac{\Sigma(X_i - \overline{\overline{X}})^2}{N-1}}$$

One can also use: $\hat{\sigma} = \frac{\overline{R}}{d_2}$ or $\tilde{\sigma} = \frac{\overline{s}}{c_4}$ as an estimate of σ in each of the theoretical formulas.

- **Probability**
  - Hypergeometric

$$Pr(X = x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

for $x = 0, 1, ..., min(n, M)$, where

$$\binom{M}{x} = \frac{M!}{x!(M-x)!} = \frac{M(M-1)\cdots(M-x+1)}{x(x-1)\cdots 1}$$

  - Binomial

$$Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

for $x = 0, 1, \cdots, n$

- **Inference**
  - Without Replacement
    * 95% upper bound on the number of defectives in the frame, M, when X=0 is found by solving the equation.

$$1 - \frac{\left(\begin{array}{c} M \\ 0 \end{array}\right)\left(\begin{array}{c} N-M \\ n \end{array}\right)}{\left(\begin{array}{c} N \\ n \end{array}\right)} \geq 0.95$$

    * 95% confidence interval on the proportion defective, p. $\hat{p} = \frac{X}{n}$ with std. error of $\hat{p} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}\left(\frac{N-n}{N}\right)}$

$$\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}\left(\frac{N-n}{N}\right)}$$

  - With Replacement
    * 95% upper bound on the proportion defective in the frame, p, when X=0.

$$p \leq 1 - (0.05)^{\frac{1}{n}}$$

    * 95% confidence interval for the proportion defective, p. $\hat{p} = \frac{X}{n}$ with std. error of $\hat{p} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$$\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$