Formulas for Final Exam

- **Process Capability**

<table>
<thead>
<tr>
<th>Theoretical</th>
<th>Estimate</th>
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</thead>
<tbody>
<tr>
<td>$C_p = \frac{USL - LSL}{6\sigma}$</td>
<td>$\hat{C}_p = \frac{USL - LSL}{6s}$</td>
</tr>
<tr>
<td>$CPU = \frac{USL - \mu}{3\sigma}$</td>
<td>$\hat{CPU} = \frac{USL - \overline{X}}{3s}$</td>
</tr>
<tr>
<td>$CPL = \frac{\mu - LSL}{3\sigma}$</td>
<td>$\hat{CPL} = \frac{\overline{X} - LSL}{3s}$</td>
</tr>
</tbody>
</table>

$C_{pk} = \min(CPU, CPL)$  \quad $\hat{C}_{pk} = \min(\hat{CPU}, \hat{CPL})$

$m = \frac{LSL + USL}{2}$  \quad $m = \frac{LSL + USL}{2}$

$k = \frac{2|m - \mu|}{(USL - LSL)}$  \quad $\hat{k} = \frac{2|m - \overline{X}|}{(USL - LSL)}$

$C_{pk} = C_p(1 - k)$  \quad $\hat{C}_{pk} = \hat{C}_p(1 - \hat{k})$

\[ s = \sqrt{\frac{\sum(X_i - \overline{X})^2}{N - 1}} \]

One can also use: $\hat{\sigma} = \frac{R}{d_2}$ or $\hat{\sigma} = \frac{s}{c_4}$ as an estimate of $\sigma$ in each of the theoretical formulas.
• Probability
  - Hypergeometric
    \[ Pr(X = x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}} \]
    for \( x = 0, 1, \ldots, \min(n, M) \), where
    \[ \binom{M}{x} = \frac{M!}{x!(M-x)!} = \frac{M(M-1)\cdots(M-x+1)}{x(x-1)\cdots1} \]
  - Binomial
    \[ Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \]
    for \( x = 0, 1, \ldots, n \)

• Inference
  - Without Replacement
    * 95% upper bound on the number of defectives in the frame, \( M \), when \( X=0 \) is found by solving the equation.
      \[ 1 - \binom{M}{0} \binom{N-M}{n} \geq 0.95 \]
    * 95% confidence interval on the proportion defective, \( p \). \( \hat{p} = \frac{X}{n} \) with std. error of \( \hat{p} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \left( \frac{N-n}{N} \right) \)
      \[ \hat{p} \pm 2 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \left( \frac{N-n}{N} \right) \]
  - With Replacement
    * 95% upper bound on the proportion defective in the frame, \( p \), when \( X=0 \).
      \[ p \leq 1 - (0.05)^\frac{1}{n} \]
    * 95% confidence interval for the proportion defective, \( p \). \( \hat{p} = \frac{X}{n} \) with std. error of \( \hat{p} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \)
      \[ \hat{p} \pm 2 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \]