

**INSTRUCTIONS:** You will have 1 hour and 30 minutes to complete the exam. There are 8 questions worth a total of 100 points. Not all questions have the same point value so gauge your time appropriately. Read the questions carefully and completely. Answer each question and show work in the space provided on the exam. Turn in the **entire** exam when you are done or when time is up. For essay questions, think before you write.

1. [10 pts] An electrical device goes through 100% inspection. Each hour's production is tested to see if it works or is defective. Because of variability in the speed of the production line, the number of devices produced varies. The data on the number of devices produced and the number defective is given below.

Hour	Number Produced	Number Defective	Hour	Number Produced	Number Defective
1	223	4	11	233	24
2	209	9	12	274	30
3	209	3	13	262	77
4	175	2	14	221	5
5	208	1	15	175	20
6	216	1	16	263	27
7	286	5	17	237	5
8	256	5	18	295	23
9	255	14	19	259	32
10	187	1	20	278	8
		2224			45
					2497
					251

- (a) [2] What is the average fraction defective for these 20 days?
- (b) [4] Is hour number 12 in or out of control? Justify your answer statistically.
- (c) [4] Is hour number 15 in or out of control? Justify your answer statistically.

2. [10 pts] The two basic principles of statistical thinking are:

- Variability always is and always will be present.
- Iterative nature of learning.

Briefly explain how each of these principles is incorporated into the construction and use of control charts.

3. [10 pts] For each of the following briefly explain whether the control charts are being used for analytic or enumerative purposes.

(a) [5] Each month a quality control engineer accesses data that are automatically collected and stored by a data acquisition computer. The engineer produces control charts that are displayed on a bulletin board.

(b) [5] Control charts with limits computed based on past data are placed in easy access of a machine operator. Periodically the operator selects a subgroup of items produced by the machine. Subgroup statistics are plotted on the control charts. If points fall outside the control limits the operator stops the machine and looks for a possible special cause.

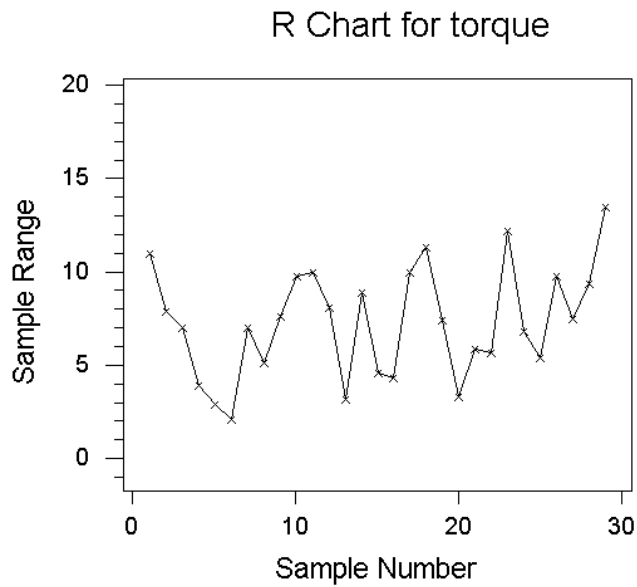
4. [20 pts] In a study performed by Shelly Kepczynski of General Motors, the torque (in Newton\*meters) on power steering pump bolts was measured using a GSE Transducerized Static Torque Wrench. Data on 29 subgroups, each of size n=5, are summarized below.

Subgroup	$\bar{X}$	$R$	Subgroup	$\bar{X}$	$R$	Subgroup	$\bar{X}$	$R$
1	37.64	11.0	11	37.04	10.0	21	38.68	5.9
2	39.00	7.9	12	38.34	8.1	22	36.82	5.7
3	39.76	7.0	13	40.58	3.2	23	38.62	12.2
4	33.58	3.9	14	39.14	8.9	24	38.20	6.8
5	40.78	2.9	15	36.78	4.6	25	39.56	5.4
6	39.36	2.1	16	39.32	4.3	27	38.10	9.8
7	39.38	7.0	17	37.20	10.0	27	38.10	7.5
8	41.14	5.1	18	34.76	11.3	28	37.52	9.4
9	39.22	7.6	19	36.82	7.4	29	34.80	13.5
10	38.10	9.8	20	38.88	3.3			

$$\bar{\bar{X}} = 38.18 \quad \bar{R} = 7.297$$

- (a) [4] Calculate the upper and lower control limits for an  $R$  chart for these data.

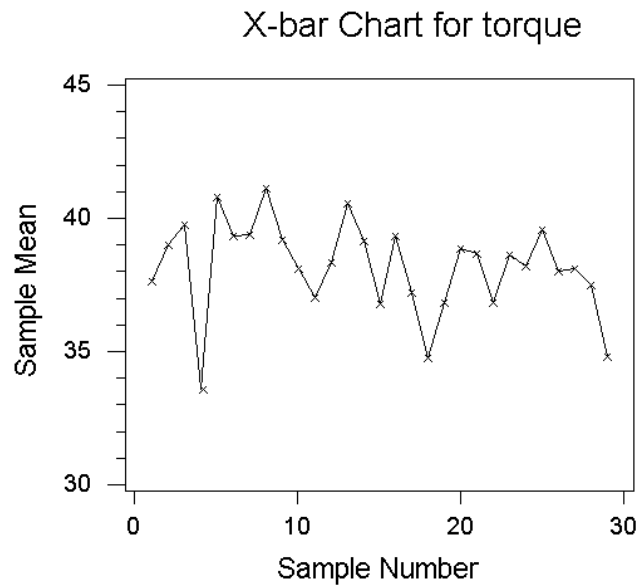
- (b) [2] Plot the control limits and a center line on the run chart below.



(c) [4] Using the one point outside control limits as our operational definition of instability, what does this  $R$  chart tell you about the stability of the process? Be specific.

(d) [4] Calculate the upper and lower control limits for an  $\bar{X}$  chart.

(e) [2] Plot the control limits and a centerline on the run chart below.



(f) [4] Using the one point outside control limits as our operational definition of instability, what does this  $\bar{X}$  chart tell you about the stability of the process? Be specific.



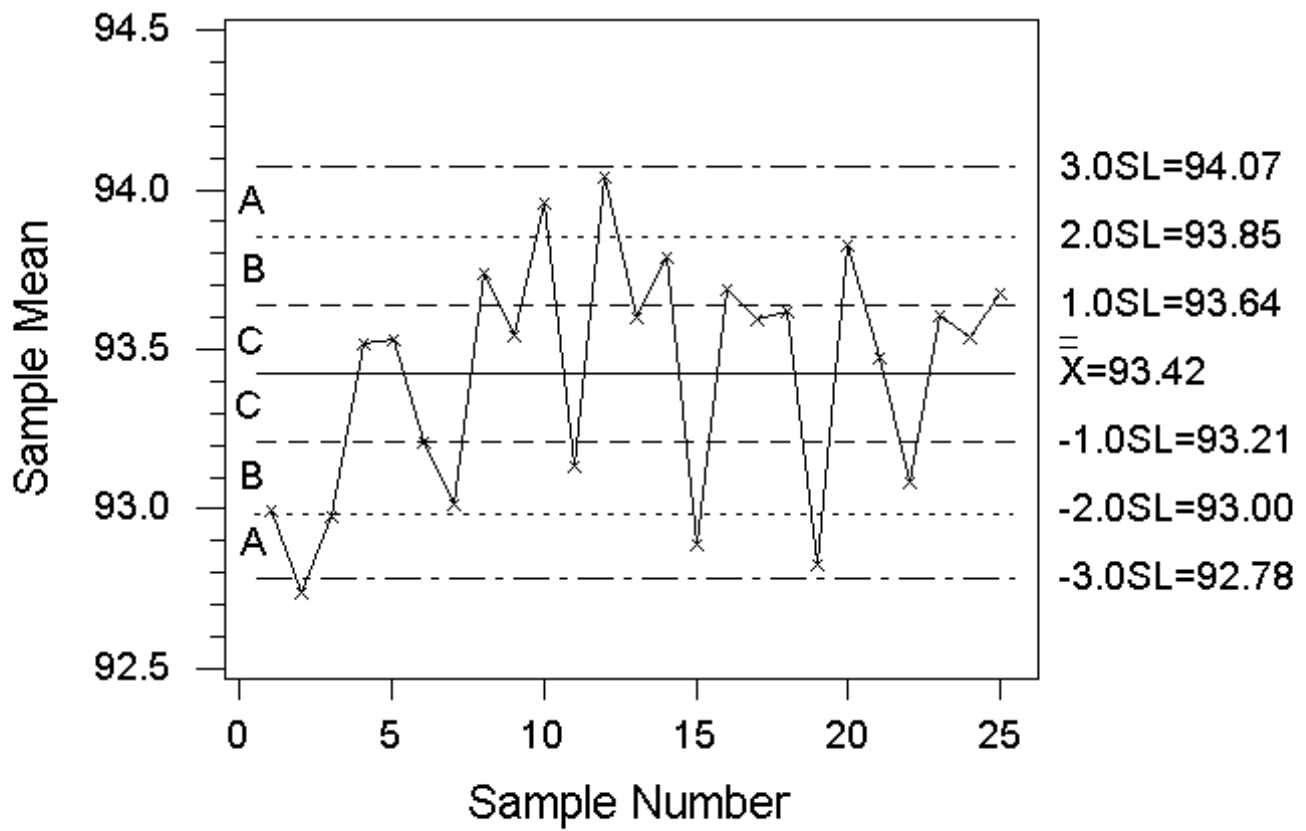
7. [15 pts] In a study performed by Ron Beals of General Motors, the amount of chrome plating on intake valves was measured. One measure was the length (in millimeters) of chrome plating on the valve stem. Subgroup means for 25 subgroups, each of size  $n=4$  are plotted on the  $\bar{X}$  chart given on the next page.

(a) [5]  $\bar{s}$  is used to establish the limits for the  $\bar{X}$  chart on the next page. What are the value of  $\bar{s}$  and the estimate of the process standard deviation based on  $\bar{s}$ ?

(b) [10] The  $\bar{X}$  chart on the next page has the center lines and 1, 2 and 3 “sigma” limits. For each of Nelson’s 8 rules (listed below) indicate whether or not there are alarms. If there are alarms, indicate at which subgroups the alarms sound.

Alarm rule: Test for special cause	Alarm?	Where?
1. One point beyond zone A (on one side of the center line)		
2. 9 points in a row in Zone C or beyond (on one side of the center line)		
3. 6 points in a row steadily increasing or decreasing		
4. 14 points in a row alternating up and down		
5. 2 of 3 points in a row in Zone A or beyond (on one side of the center line)		
6. 4 of 5 points in a row in Zone B or beyond (on one side of the center line)		
7. 15 points in a row in Zone C (above and below the center line)		
8. 8 points in a row with none in Zone C (above and below the center line)		

# X-bar Chart for Length



Test continues on the next page

8. [15 pts] We have seen that the test that sounds an alarm when a single point plots outside 3 “sigma” limits has an  $ARL = \frac{1}{r}$  where

$$r = Pr(\bar{X} > \mu + 3\frac{\sigma}{\sqrt{n}}) + Pr(\bar{X} < \mu - 3\frac{\sigma}{\sqrt{n}})$$

It can be shown that an alternative test that sounds an alarm when the **second** point plots outside  $m$  “sigma” limits has an  $ARL = \frac{2}{q}$  where

$$q = Pr(\bar{X} > \mu + m\frac{\sigma}{\sqrt{n}}) + Pr(\bar{X} < \mu - m\frac{\sigma}{\sqrt{n}})$$

- (a) [5] Using this alternative test with  $m = 2$ , what is the ARL for a process that is normally distributed with center at  $\mu$  and spread  $\sigma$  (*i.e.* an in control on target process)?

- (b) [5] What would  $m$  have to be in order for this alternative test to have an ARL of 385 when the process is in control and on target?

- (c) [5] Using subgroups of size  $n=4$  and the alternative test with  $m = 2$ , what is the ARL for a process that is normally distributed but has shifted  $1\sigma$  from the center  $\mu$ ?