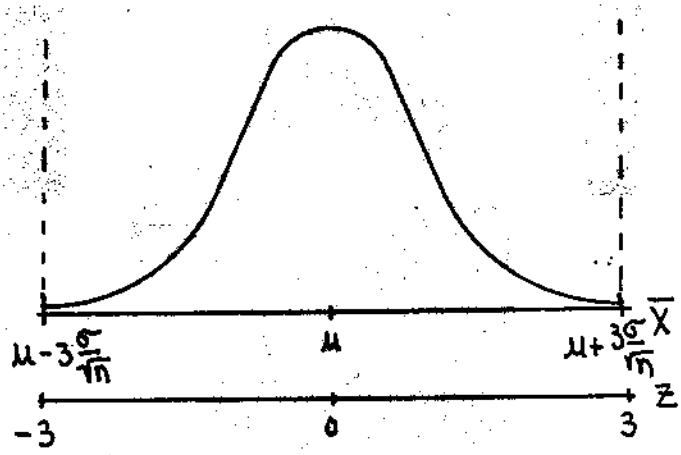
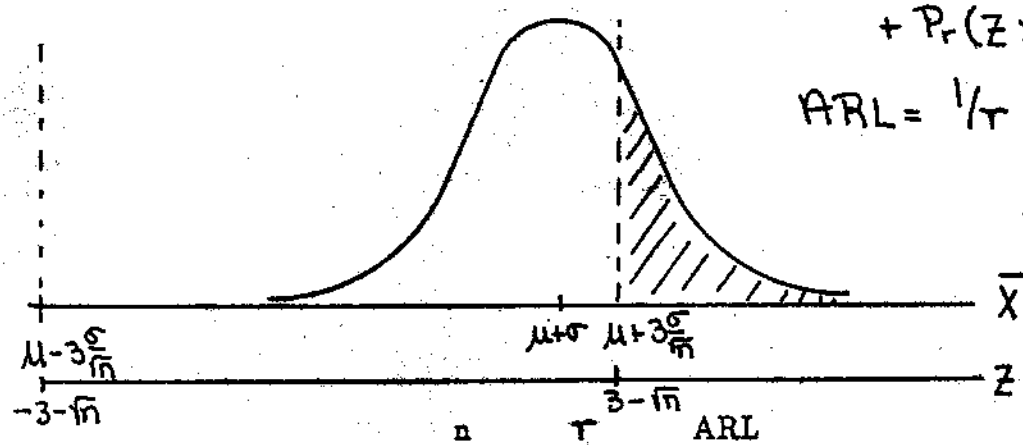
Theoretical Calculation of Average Run Length
Shift = 1σ



$$r = P_r(Z < -3 - \sqrt{n}) + P_r(Z > 3 - \sqrt{n})$$

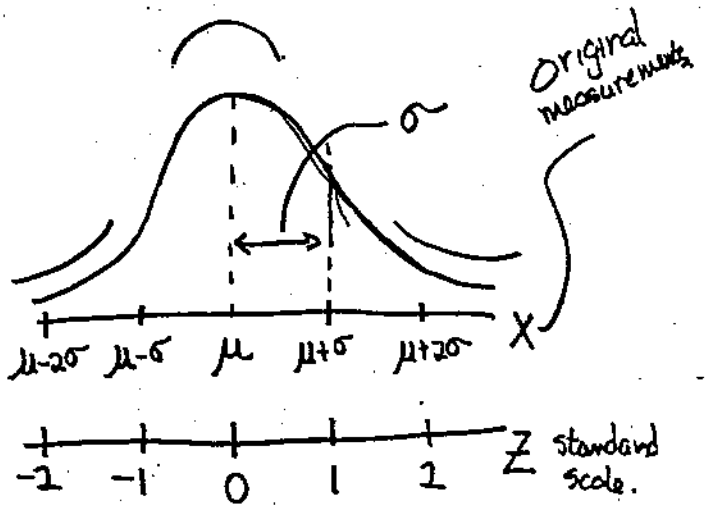
$$ARL = 1/r$$



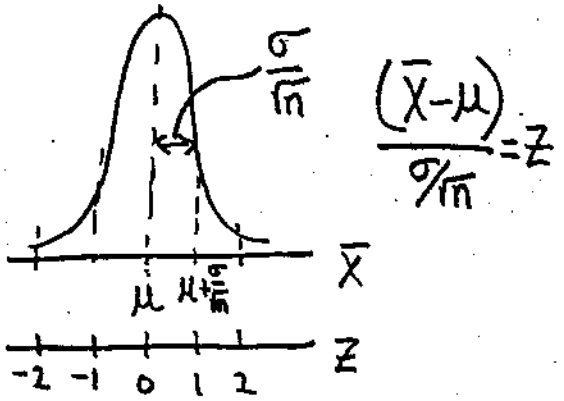
n	r	ARL
2	0.056	17.9
3	0.102	9.8
4	0.159	6.3
5	0.222	4.5
6	0.291	3.4
7	0.362	2.8
8	0.432	2.3
9	0.500	2.0
10	0.564	1.8
15	0.809	1.2
20	0.930	1.1
25	0.977	1.0

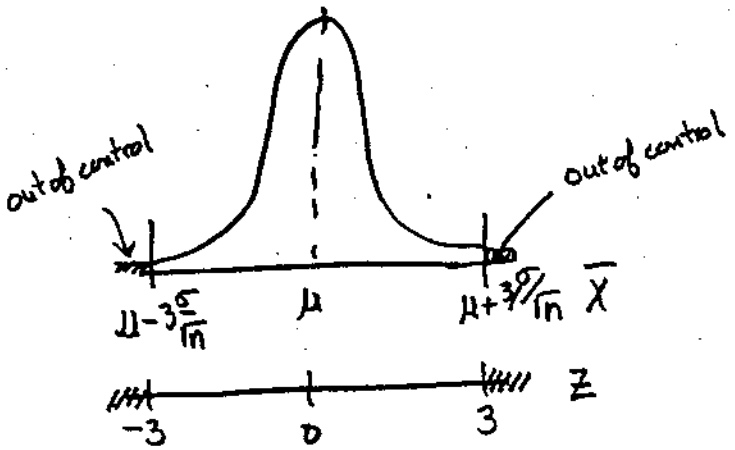
Standard Normal Distribution

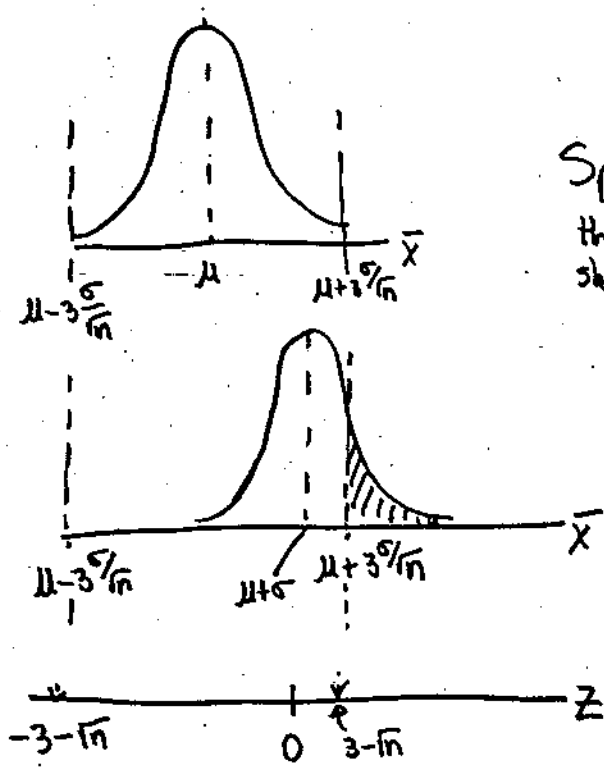
z-value	area to left of z-value	area to right of z-value	z-value	area to left of z-value	area to right of z-value
-3.4	.0003	.9997	0.0	.5000	.5000
-3.3	.0005	.9995	0.1	.5398	.4602
-3.2	.0007	.9993	0.2	.5793	.4207
-3.1	.0010	.9990	0.3	.6179	.3821
-3.0	.0013	.9987	0.4	.6554	.3446
-2.9	.0019	.9981	0.5	.6915	.3085
-2.8	.0026	.9974	0.6	.7257	.2743
-2.7	.0035	.9965	0.7	.7580	.2420
-2.6	.0047	.9953	0.8	.7881	.2119
-2.5	.0062	.9938	0.9	.8159	.1841
-2.4	.0082	.9918	1.0	.8413	.1587
-2.3	.0107	.9893	1.1	.8643	.1357
-2.2	.0139	.9861	1.2	.8849	.1151
-2.1	.0179	.9821	1.3	.9032	.0968
-2.0	.0228	.9772	1.4	.9192	.0808
-1.9	.0287	.9713	1.5	.9332	.0668
-1.8	.0359	.9641	1.6	.9452	.0548
-1.7	.0446	.9554	1.7	.9554	.0446
-1.6	.0548	.9452	1.8	.9641	.0359
-1.5	.0668	.9332	1.9	.9713	.0287
-1.4	.0808	.9192	2.0	.9772	.0228
-1.3	.0968	.9032	2.1	.9821	.0179
-1.2	.1151	.8849	2.2	.9861	.0139
-1.1	.1357	.8643	2.3	.9893	.0107
-1.0	.1587	.8413	2.4	.9918	.0082
-0.9	.1841	.8159	2.5	.9938	.0062
-0.8	.2119	.7881	2.6	.9953	.0047
-0.7	.2420	.7580	2.7	.9965	.0035
-0.6	.2743	.7257	2.8	.9974	.0026
-0.5	.3085	.6915	2.9	.9981	.0019
-0.4	.3446	.6554	3.0	.9987	.0013
-0.3	.3821	.6179	3.1	.9990	.0010
-0.2	.4207	.5793	3.2	.9993	.0007
-0.1	.4602	.5398	3.3	.9995	.0005
0.0	.5000	.5000	3.4	.9997	.0003



random samples of size n.







Spoz:
that the distribution shifts 1 σ.

$$\frac{\left(UCL-(\mu+\sigma)\right)}{\sigma/\sqrt{n}} = \frac{\mu+3\frac{\sigma}{\sqrt{n}}-(\mu+\sigma)}{\sigma/\sqrt{n}}$$

$$= \frac{\left(3\frac{\sigma}{\sqrt{n}}-\sigma\right)\left(\frac{\sqrt{n}}{\sigma}\right)}{\frac{\sigma}{\sqrt{n}}\left(\frac{\sqrt{n}}{\sigma}\right)}$$

$$= 3-\sqrt{n}$$

$$r = Pr(Z < -3-\sqrt{n}) + Pr(Z > 3-\sqrt{n})$$

$$ARL = \frac{1}{r}$$