Stat 432: Homework 7

1. Below is a maze with 7 compartments.

   A mouse is put into the maze in one of the compartments 1, 2, 4, 5, or 6. The mouse moves through compartments at random, that is if there are \( n \) ways to leave a compartment the mouse chooses each with probability \( 1/n \). Assume that once the mouse reaches compartment 3 (Food) or compartment 7 (Shock), the mouse is removed from the maze. That is, these two compartments can be treated as absorbing states because once in one of these compartments the mouse can no longer move to another compartment because it is removed from the maze.

   a) Construct the single-step transition probability matrix for the seven states.
   b) Find the probability that if the mouse starts in compartment 4, the mouse finds the food (3).
   c) Find the probability that if the mouse starts in compartment 6, the mouse finds the food (3).
   d) Find the expected time until the mouse reaches either compartment 3 (Food) or 7 (Shock) given the mouse starts in compartment 5.

2. Use the same maze as in problem 1 only this time only compartment 3 (Food) is an absorbing state.

   a) Construct the single-step transition probability matrix for the seven states.
   b) Find the probability that if the mouse starts in compartment 4, the mouse finds the food (3).
   c) Find the probability that if the mouse starts in compartment 6, the mouse finds the food (3).
   d) Find the expected time until the mouse reaches either compartment 3 (Food) given the mouse starts in compartment 5.

3. In problem 1 of homework 6, we looked at a model for social mobility. Using the one-step transition probabilities from that problem find the limiting probabilities for the three states Professional, Skilled Worker and Unskilled Worker.

4. Consider the one-step transition probabilities for the social mobility model given below. Using these probabilities find the limiting probabilities for the three states P, S and U.

\[
\begin{array}{c|c|c|c}
 & P & S & U \\
\hline
P & 0.60 & 0.20 & 0.20 \\
S & 0.25 & 0.50 & 0.25 \\
U & 0.15 & 0.30 & 0.55 \\
\end{array}
\]