Stat 432: Homework 3

1. The number of telephone calls coming into a switchboard during a 10 minute period is a Poisson random variable with an average rate of 4.
   a. What is the probability that exactly 1 call comes in during a 10 minute period?
   b. Exactly 1 call comes in during a 10 minute period. What is the probability that the call comes in during the first 2 minutes?
   c. Exactly 1 call comes in during a 10 minute period. What is the expected time for that call?

2. The proportion of time per hour that all checkout counters in a supermarket are busy is a random variable, $X$, with density function given below.

$$f(x) = cx(1-x)^2 \quad 0 < x < 1$$

= 0 \quad \text{otherwise}

a. Find $c$.
   b. Give the distribution function for $X$, e.g. $F(x)$.
   c. What is the probability that more than 60% of the time all checkout counters are busy?
   d. What are $E(X)$ and $Var(X)$?
   e. Verify that for this random variable $E(X) = \int_0^1 [1-F(x)]dx$

3. One-hour carbon monoxide concentrations in air samples in a small city are modeled with an exponential random variable with mean 2 parts per million.
   a. What is the probability that the one-hour carbon monoxide concentration exceeds 5 parts per million?
   b. What is the probability that the one-hour carbon monoxide concentration is within one standard deviation of the mean?

4. Let $X$ be an exponentially distributed random variable with parameter $\beta$. Define a new random variable

$$Y = k \text{ if } k - 1 \leq X < k \text{ for } k = 1, 2, 3, 4, \ldots$$

a. Find $Pr(Y = k)$.
   b. The random variable $Y$ is a common discrete random variable. What discrete random variable is it? Be specific.