One-sample location problem

- **Assumptions:**
  - Observations are independent.
  - Observations come from a population with a continuous distribution function with population median η.

- **Testing:** $H : \eta = \eta_o$**, sign test**

Alternative	Test Statistic	P-value	Approximate P-value
$A : \eta > \eta_o$	S_+	Table F: P for Right S	$\Pr\left(Z \geq \frac{S_+ - 0.5 - \frac{n}{2}}{\frac{\sqrt{n}}{2}}\right)$
$A : \eta < \eta_o$	S_-	Table F: P for Right S	$\Pr\left(Z \geq \frac{S_- - 0.5 - \frac{n}{2}}{\frac{\sqrt{n}}{2}}\right)$
$A : \eta \neq \eta_o$	S = Larger(S_+,S_-)	Table F: twice P for Right S	$2\Pr\left(Z \geq \frac{S - 0.5 - \frac{n}{2}}{\frac{\sqrt{n}}{2}}\right)$

- **Estimation: ordered observations**

Point estimate of η is the sample median M.

Confidence interval for η goes from $X_{(k+1)}$ to $X_{(n-k)}$, where k is found using Table F with a P corresponding to (1 - γ)/2. Where γ is the level of confidence.

Table F	Left		Right
n	S	P	S
n	k	$\frac{1-\gamma}{2}$	n-k

For large values of n,

$$k = \frac{n}{2} - 0.5 - z\frac{\sqrt{n}}{2}$$

where z corresponds to the standard normal z value for the given level of confidence.

- **Assumptions:**
  - Observations are independent.
  - Observations come from a population with a continuous distribution function symmetric about the population median η.
- **Testing:** $H : \eta = \eta_o$, **Wilcoxon signed-rank test**

Alternative	Test Statistic	P-value	Approximate P-value
$A : \eta > \eta_o$	T_+	Table G: P for Right T	$\Pr\left(Z \geq \frac{T_+ - 0.5 - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}\right)$
$A : \eta < \eta_o$	T_-	Table G: P for Right T	$\Pr\left(Z \geq \frac{T_- - 0.5 - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}\right)$
$A : \eta \neq \eta_o$	T = Larger(T_+,T_-)	Table G: twice P for Right T	$2\Pr\left(Z \geq \frac{T - 0.5 - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}\right)$

In the case of tied ranks, Table G does not give the exact null distribution for the test statistic. For the normal approximation, the standard deviation should be reduced by taking into account the size of the tied groups, t_i.

$$\sqrt{\frac{n(n+1)(2n+1)}{24} - \frac{\sum t_i(t_i^2 - 1)}{48}}$$

- **Estimation: ordered Walsh averages**

Point estimate of η is the median of the n(n+1)/2 Walsh averages, U_i.

Confidence interval for η goes from $U_{(k+1)}$ to $U_{(n-k)}$, where k is found using Table G with a P corresponding to (1 - γ)/2. Where γ is the level of confidence.

Table G	Left		Right
n	T	P	T
n	k	$\frac{1-\gamma}{2}$	n-k

For large values of n,

$$k = \frac{n(n+1)}{4} - 0.5 - z\sqrt{\frac{n(n+1)(2n+1)}{24}}$$

where z corresponds to the standard normal z value for the given level of confidence.