Regression Example

<table>
<thead>
<tr>
<th>R(X)</th>
<th>X</th>
<th>Y</th>
<th>R(Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.3</td>
<td>1.4</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4.1</td>
<td>5.3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5.6</td>
<td>4.8</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>7.2</td>
<td>6.5</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>9.2</td>
<td>11.0</td>
<td>5</td>
</tr>
</tbody>
</table>

Regression of $Y$ on $X$

- **Least Squares Regression**
  
slope estimate: $\hat{\beta}_1 = r \left( \frac{s_Y}{s_X} \right)$
  
inintercept estimate: $\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X}$

summaries: $\overline{X} = 5.68$, $s_X = 2.675$, $\overline{Y} = 5.8$, $s_Y = 3.469$, $r = 0.946$

slope estimate: $\hat{\beta}_1 = 0.946 \left( \frac{3.469}{2.675} \right) = 1.227$

intercept estimate: $\hat{\beta}_0 = 5.8 - 1.227(5.68) = -1.169$

equation: $\hat{Y} = -1.17 + 1.23X$

prediction: if $X=6.2$ then $\hat{Y} = -1.17 + 1.23(6.2) = 6.46$
• Rank Regression

rank slope estimate: ls slope estimate calculated on ranks
rank intercept estimate: ls intercept estimate calculated on ranks

summarizes of ranks: \( \bar{R}(X) = 3.0, s_{R(X)} = 1.581, \bar{R}(Y) = 3.0, s_{R(Y)} = 1.581, r_S = 0.9 \)
rank slope estimate: rank \( \hat{\beta}_1 = 0.9 \left( \frac{1.581}{1.581} \right) = 0.9 \)
rank intercept estimate: rank \( \hat{\beta}_0 = 3.0 - 0.9(3.0) = 0.3 \)
rank equation: \( \hat{R}(Y) = 0.3 + 0.9\hat{R}(X) \)
prediction: if \( X = 6.2 \), then \( \hat{R}(X) = 3.375 \)
\[
\hat{Y} = 5.3 + [3.3375 - 3][6.5 - 5.3] = 5.3 + .3375[1.2] = 5.71
\]

• Theil-Sen-Adichie

Consider all pairs of points \((X_i, Y_i)\) \((X_j, Y_j)\) for \(1 \leq i < j \leq n\) and the associated elementary slope estimates.
\[
S_{ij} = \frac{Y_j - Y_i}{X_j - X_i}
\]
slope estimate: \( \tilde{\beta}_1 = \text{median } S_{ij} \)
intercept estimate: \( \tilde{\beta}_0 = \text{median } Y - \tilde{\beta}_1 \text{ median } X \)

| \(\begin{array}{cccc}
  i=1 & i=2 & i=3 & i=4 \\
  j=2 & 2.17 & & \\
  j=3 & 1.03 & -0.33 & \\
  j=4 & 1.04 & 0.39 & 1.06 \\
  j=5 & 1.39 & 1.12 & 1.72 & 2.25 \\
\end{array}\) |

summarizes: median \(X = 5.6\), median \(Y = 5.3\), \(T = 0.8\)
slope estimate: \( \tilde{\beta}_1 = \frac{1.06+1.12}{2} = 1.09 \)
intercept estimate: \( \tilde{\beta}_0 = 5.3 - 1.09(5.6) = -0.8 \)
equation: \( \tilde{Y} = -0.8 + 1.09X \)
prediction: if \(X=6.2\) then \( \tilde{Y} = -0.8 + 1.09(6.2) = 5.96 \)