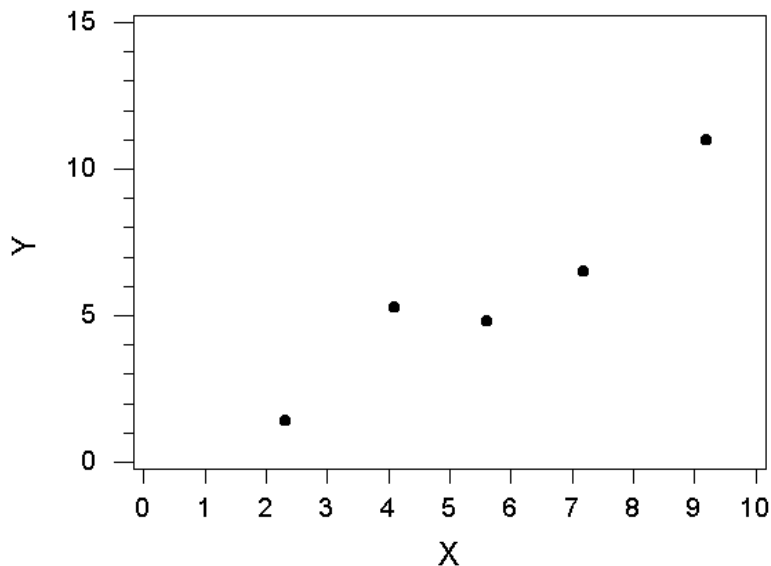


Regression Example

R(X)	X	Y	R(Y)
1	2.3	1.4	1
2	4.1	5.3	3
3	5.6	4.8	2
4	7.2	6.5	4
5	9.2	11.0	5

Regression of Y on X



- **Least Squares Regression**

slope estimate: $\hat{\beta}_1 = r \left(\frac{s_Y}{s_X} \right)$

intercept estimate: $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$

summaries: $\bar{X} = 5.68$, $s_X = 2.675$, $\bar{Y} = 5.8$, $s_Y = 3.469$, $r = 0.946$

slope estimate: $\hat{\beta}_1 = 0.946 \left(\frac{3.469}{2.675} \right) = 1.227$

intercept estimate: $\hat{\beta}_0 = 5.8 - 1.227(5.68) = -1.169$

equation: $\hat{Y} = -1.17 + 1.23X$

prediction: if $X=6.2$ then $\hat{Y} = -1.17 + 1.23(6.2) = 6.46$

- **Rank Regression**

rank slope estimate: ls slope estimate calculated on ranks

rank intercept estimate: ls intercept estimate calculated on ranks

summaries of ranks: $\overline{R(X)} = 3.0$, $s_{R(X)} = 1.581$, $\overline{R(Y)} = 3.0$, $s_{R(Y)} = 1.581$, $r_S = 0.9$

rank slope estimate: $\text{rank}\hat{\beta}_1 = 0.9 \left(\frac{1.581}{1.581} \right) = 0.9$

rank intercept estimate: $\text{rank}\hat{\beta}_0 = 3.0 - 0.9(3.0) = 0.3$

rank equation: $R(\hat{Y}) = 0.3 + 0.9R(X)$

prediction: if $X = 6.2$, then $R(X) = 3.375$

$$R(\hat{Y}) = 0.3 + 0.9(3.375) = 3.3375$$

$$\tilde{Y} = 5.3 + [3.3375 - 3][6.5 - 5.3] = 5.3 + .03375[1.2] = 5.71$$

- **Theil-Sen-Adichie**

Consider all pairs of points (X_i, Y_i) (X_j, Y_j) for $1 \leq i < j \leq n$ and the associated elementary slope estimates.

$$S_{ij} = \frac{Y_j - Y_i}{X_j - X_i}$$

slope estimate: $\tilde{\beta}_1 = \text{median } S_{ij}$

intercept estimate: $\tilde{\beta}_0 = \text{median } Y - \tilde{\beta}_1 \text{ median } X$

	i=1	i=2	i=3	i=4
j=2	2.17			
j=3	1.03	-0.33		
j=4	1.04	0.39	1.06	
j=5	1.39	1.12	1.72	2.25

summaries: median X = 5.6, median Y = 5.3, T = 0.8

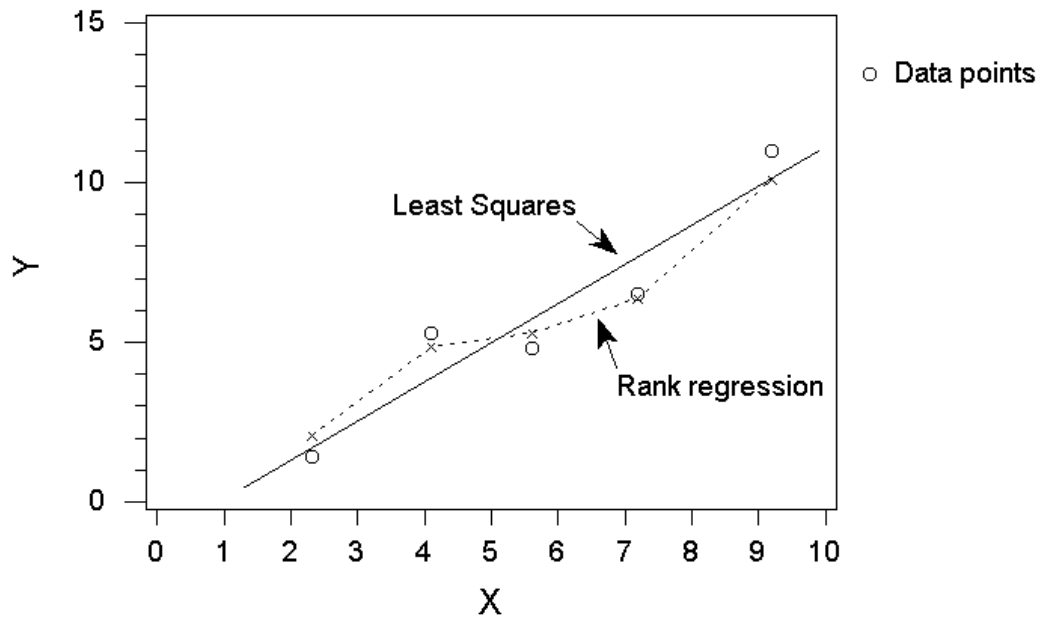
slope estimate: $\tilde{\beta}_1 = \frac{1.06+1.12}{2} = 1.09$

intercept estimate: $\tilde{\beta}_0 = 5.3 - 1.09(5.6) = -0.8$

equation: $\tilde{Y} = -0.8 + 1.09X$

prediction: if $X=6.2$ then $\tilde{Y} = -0.8 + 1.09(6.2) = 5.96$

Plot of Y versus X with prediction equations



Plot of Y versus X with prediction equations

