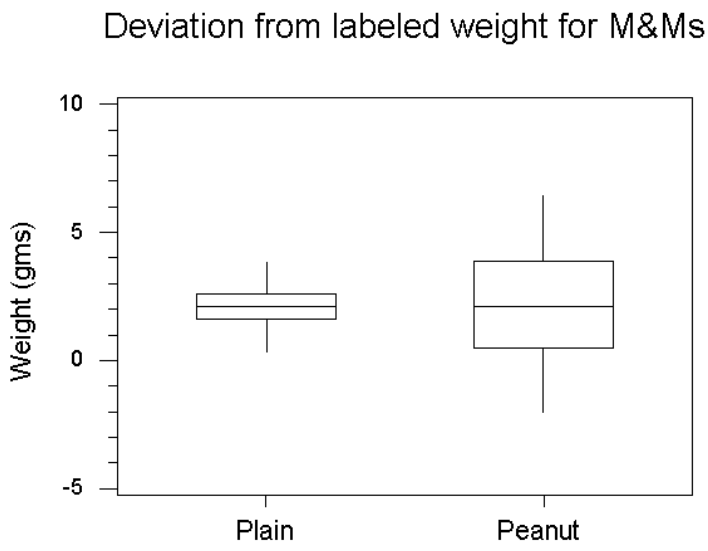
Stat 403 - Solution to Assignment 7

Turned in Thursday, November 9, 2000

1. M&Ms are packaged in small packages for individual sale in vending machines. Each package has a labeled weight. The actual weight of contents may differ from the labeled weight due to variability in the packaging process or the product being packaged. Below are deviations from the labeled weight (grams) for 15 small bags of Plain M&Ms and 15 small bags of Peanut M&Ms. A positive deviation indicates that the contents of the bag weighed more than the labeled weight. A negative deviation indicates that the contents of the bag weighed less than the labeled weight.

Plain M&Ms					Peanut M&Ms				
2.6	1.7	0.3	2.0	2.8	1.0	3.9	5.8	−0.1	6.5
2.6	1.0	1.6	1.5	2.6	2.7	1.9	3.1	0.5	2.1
1.6	2.1	2.3	3.9	2.5	4.6	−1.3	2.6	−2.1	1.6

(a) Side-by-side boxplots of the deviations.



The deviations from the labeled weight for Peanut M&Ms tend to be more spread out than those for Plain M&Ms. The centers are about equal and both distributions look symmetric (maybe even normal).

(b) Calculation of the Siegel-Tukey test statistic appears on the next page. The two-sided P-value is 0.0192. This small a value suggests that the two distributions differ in terms of scale (spread). If we had suspected that Peanut M&Ms would have been more variable (peanuts are naturally more variable than the chocolate center in a Plain M&M) we would have done a one-sided test.

Plain, X	Peanut, Y	Rank	Tied Rank	S-T_X	S-T_Y
	−2.1	29			29
	−1.3	28			28
	−0.1	25			25
0.3		24		24	
	0.5	21			21
1.0	**1.0**	20,17	18.5	18.5	18.5
1.5		16		16	
1.6	**1.6**	13,12	11.3	11.3	11.3
1.6		9	11.3	11.3	
1.7		8		8	
	1.9	5			5
2.0		4		4	
2.1	**2.1**	1,2	1.5	1.5	1.5
2.3		3		3	
2.5		6		6	
2.6		7	10.5	10.5	
2.6	**2.6**	11,10	10.5	10.5	10.5
2.6		14	10.5	10.5	
	2.7	15			15
2.8		18		18	
	3.1	19			19
3.9	**3.9**	23,22	22.5	22.5	22.5
	4.6	26			26
	5.8	27			27
	6.5	30			30
				175.6	289.3

$U_X = 175.6 - \frac{15(16)}{2} = 55.6 \quad U_Y = 289.3 - \frac{15(16)}{2} = 169.3$

mean of U $= \frac{mn}{2} = \frac{15(15)}{2} = 112.5$

std dev of U $= \sqrt{\frac{mn(m+n+1)}{12}} = \sqrt{\frac{15(15)(31)}{12}} = \sqrt{581.25} = 24.11$

std dev of U (corrected for ties) $= \sqrt{\frac{mn}{12}\left[(m+n+1) - \frac{\sum t_j(t_j^2-1)}{(m+n)(m+n-1)}\right]}$

$= \sqrt{\frac{15(15)}{12}\left[(15+15+1) - \frac{2(3)+3(8)+2(3)+4(15)+2(3)}{30(29)}\right]} = \sqrt{579.05} = 24.06$

P-value $= 2Pr(U_X \geq 169.3) \doteq 2Pr\left(Z \geq \frac{169.3-0.5-112.5}{24.11}\right) = 2Pr(Z \geq 2.34) = 2(0.0096) = 0.0192$

Since the P-value is small, there is sufficient evidence to state that the two distributions differ in terms of scale (spread).

(c) Calculation of the Fligner test statistic, combined sample median, M=2.1.

Plain, X	$\lvert X-M\rvert$	Rank	Peanut, Y	$\lvert Y-M\rvert$	Rank
0.3	1.8	23	−2.1	4.2	29
1.0	1.1	19.5	−1.3	3.4	27
1.5	0.6	15.5	−0.1	2.2	25
1.6	0.5	11	0.5	1.6	21
1.6	0.5	11	1.0	1.1	19.5
1.7	0.4	6.5	1.6	0.5	11
2.0	0.1	3	1.9	0.2	4.5
2.1	0.0	1.5	2.1	0.0	1.5
2.3	0.2	4.5	2.6	0.5	11
2.5	0.4	6.5	2.7	0.6	15.5
2.6	0.5	11	3.1	1.0	18
2.6	0.5	11	3.9	1.8	23
2.6	0.5	11	4.6	2.5	26
2.8	0.7	17	5.8	3.7	28
3.9	1.8	23	6.5	4.3	30
		Fl_X=175			Fl_Y=290

$U_X = 175 - \frac{15(16)}{2} = 55 \quad U_Y = 290 - \frac{15(16)}{2} = 170$

mean of U $= \frac{mn}{2} = \frac{15(15)}{2} = 112.5$

std dev of U $= \sqrt{\frac{mn(m+n+1)}{12}} = \sqrt{\frac{15(15)(31)}{12}} = \sqrt{581.25} = 24.11$

std dev of U (corrected for ties) $= \sqrt{\frac{mn}{12}\left[(m+n+1) - \frac{\sum t_j(t_j^2-1)}{(m+n)(m+n-1)}\right]}$

$= \sqrt{\frac{15(15)}{12}\left[(15+15+1) - \frac{2(3)+2(3)+2(3)+7(48)+2(3)+2(3)+3(8)}{30(29)}\right]} = \sqrt{572.84} = 23.93$

P-value $= 2Pr(U_X \geq 170) \doteq 2Pr\left(Z \geq \frac{170-0.5-112.5}{23.93}\right) = 2Pr(Z \geq 2.38) = 2(0.0087) = 0.0174$. If you do not correct for ties, the P-value is 0.0182.

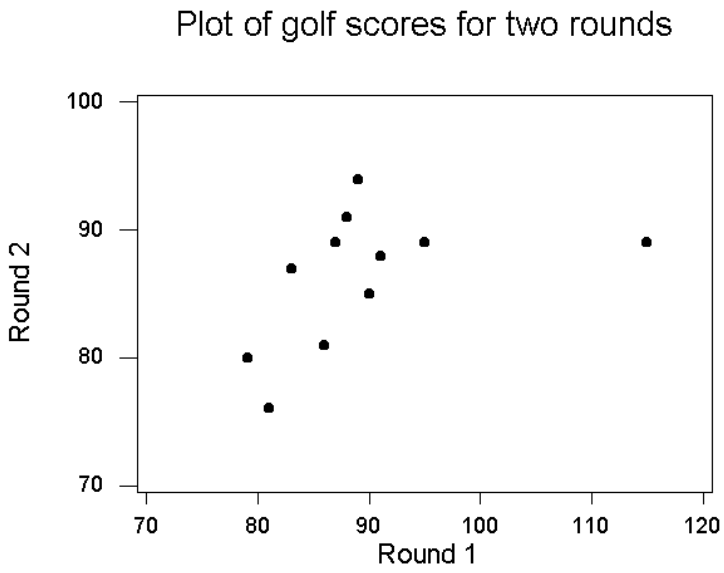
Since the P-value is small, there is sufficient evidence to state that the two distributions differ in terms of scale (spread).

(d) Both procedures give about the same value of the test statistic, P-value, and conclusion.

2. Below are the scores for eleven women golfers for a tournament that consisted of two rounds of golf.

Golfer	1	2	3	4	5	6	7	8	9	10	11
Round 1, X	89	90	87	95	86	81	115	83	88	91	79
Round 2, Y	94	85	89	89	81	76	89	87	91	88	80

(a) Below is the plot of the data. In general, the golfers who have high (low) scores in the first Round tend to have high (low) scores in the second Round. Golfer #7 is an exception in that she had the highest score in the first round but had a middle score on the second round.



(b) r = 0.460, based on all 11 golfers.

(c) r = 0.661, omitting golfer #7.

(d) Spearman's rank correlation for all 11 golfers.

Golfer	1	2	3	4	5	6	7	8	9	10	11
U_i	7	8	5	10	4	2	11	3	6	9	1
Round 1, X_i	89	90	87	95	86	81	115	83	88	91	79
Round 2, Y_i	94	85	89	89	81	76	89	87	91	88	80
V_i	11	4	8	8	3	1	8	5	10	6	2
D_i	−4	4	−3	2	1	1	3	−2	−4	3	−1
D_i^2	16	16	9	4	1	1	9	4	16	9	1

$r_S = 1 - \frac{6\sum D_i^2}{n(n^2-1)} = 1 - \frac{6(86)}{11(120)} = 1 - 0.391 = 0.609$

(e) P-value is between 0.025 and 0.05. There is a statistically significant monotonic relationship between first and second round scores.

(f) Spearman's rank correlation, golfer #7 omitted

Golfer	1	2	3	4	5	6	7	8	9	10	11
U_i	7	8	5	10	4	2		3	6	9	1
Round 1, X_i	89	90	87	95	86	81	*	83	88	91	79
Round 2, Y_i	94	85	89	89	81	76	*	87	91	88	80
V_i	10	4	7.5	7.5	3	1		5	9	6	2
D_i	-3	4	-2.5	2.5	1	1		-2	-3	3	-1
D_i^2	9	16	6.25	6.25	1	1		4	9	9	1

$r_S = 1 - \frac{6\sum D_i^2}{n(n^2-1)} = 1 - \frac{6(62.5)}{10(99)} = 1 - 0.379 = 0.621$

(g) P-value is between 0.030 and 0.033. There is a statistically significant monotonic relationship between first and second round scores.

(h) The Pearson correlation coefficient appears to be more sensitive to unusual values. When all 11 golfers are included, the value of r is quite low and is not statistically significant. When golfer #7 is omitted, the correlation increases and becomes statistically significant.

The Spearman rank correlation is less affected by the one unusual golfer. Both values (with and without golfer #7) of r_S are about the same and both indicate a significant monotonic relationship.