Stat 403 - Solution to Assignment 5

Turned in Thursday, October 19, 2000

1. Hotpoint has a standard material for use in insulating the ovens of stoves that they manufacture. They decide to explore the use of a different material that is cheaper to purchase but may not have the same degree of insulating power. Below are the times it takes for ovens to cool down to 350°F after being heated to 400°F and the heating element turned off.

Standard Material, X	13.9	14.2	14.9	15.1	15.3	15.7	16.1	17.2
Cheaper Material, Y	12.9	13.7	14.0	14.1	14.4	14.7	14.8	15.4

(a) Summary of cool down times.

Standard Material, X	Cheaper Material, Y
m = 8	n= 8
$\overline{X} = 15.3$	$\overline{Y} = 14.25$
$s_X = 1.054$	$s_Y = 0.762$
$s_X^2 = 1.11$	$s_Y^2 = 0.58$
$s_p^2 = 0.845$	$s_p = 0.919$

H: $\mu_X = \mu_Y$
A: $\mu_X > \mu_Y$

$$t = \frac{\overline{X} - \overline{Y}}{s_p\sqrt{\frac{1}{m} + \frac{1}{n}}} = \frac{15.3 - 14.25}{0.919\sqrt{\frac{1}{8} + \frac{1}{8}}} = \frac{1.95}{0.46} = 2.28$$

df = m + n - 2 = 14

The P-value is between 0.01 and 0.02 (from Minitab, P-value = 0.019). Since the P-value is so small we reject H. These data indicate that the standard material does have a longer mean cool down time than the cheaper material.

(b) Mann-Whitney-Wilcoxon test.

Rank X	3	6	10	11	12	14	15	16	$W_X = 87$
Standard Material, X	13.9	14.2	14.9	15.1	15.3	15.7	16.1	17.2	
Cheaper Material, Y	12.9	13.7	14.0	14.1	14.4	14.7	14.8	15.4	
Rank Y	1	2	4	5	7	8	9	13	$W_Y = 49$

H: $\eta_X = \eta_Y$
A: $\eta_X > \eta_Y$

$U_X = 87 - \frac{8(9)}{2} = 51 \quad U_Y = 49 - \frac{8(9)}{2} = 13$

Table H	Left		Right
n	U	P	U
		m=8	
8	13	0.025	51

The P-value is 0.025. Since the P-value is so small we reject H. These data indicate that the standard material does have a longer mean cool down time than the cheaper material.

(c) Yes, the P-values are close. The t-test has a slightly smaller P-value (0.019) compared to the Mann-Whitney-Wilcoxon P-value (0.025).

(d) The estimate of the difference in population means is: $\overline{X} - \overline{Y} = 15.3 - 14.25 = 1.05$

A 95% confidence interval for the difference in means is:

$$\overline{X} - \overline{Y} \pm t^* s_p \sqrt{\frac{1}{m} + \frac{1}{n}} = 1.05 \pm 2.145(0.919)\sqrt{\frac{1}{8} + \frac{1}{8}} = 1.05 \pm 0.99$$

(0.06, 2.04)

(e) Table of $\Delta_{ij} = X_i - Y_j$

Y \ X	13.9	14.2	14.9	15.1	15.3	15.7	16.1	17.2
15.4	−1.5	−1.2	−0.5	−0.3	−0.1	0.3	0.7	1.8
14.8	−0.9	−0.6	0.1	0.3	0.5	0.9	1.3	2.4
14.7	−0.8	−0.5	0.2	0.4	0.6	1.0	1.4	2.5
14.4	−0.5	−0.2	0.5	0.7	0.9	1.3	1.7	2.8
14.1	−0.2	0.1	0.8	1.0	1.2	1.6	2.0	3.1
14.0	−0.1	0.2	0.9	1.1	1.3	1.7	2.1	3.2
13.7	0.2	0.5	1.2	1.4	1.6	2.0	2.4	3.5
12.9	1.0	1.3	2.0	2.2	2.4	2.8	3.2	4.3

Median of the $\Delta_{ij} = 1.0$ is the estimate of the difference in median cool down times.

Table H	Left		Right
n	U	P	U
		m=8	
8	13	0.025	51

A 95% confidence interval for the difference in median cool down times goes from the 14th to the 51st ordered Δ_{ij}

(0.10, 2.00)

(f) The confidence intervals are quite close. In this example, the Mann-Whitney-Wilcoxon interval is a bit narrower than the t-interval

(g) Analysis of the data on cool down times for the two insulating materials indicates that the cheaper material does not insulate as well as the standard material. There is about a 1 minute shorter cool down time for the cheaper material. Allowing for variability in this estimate, we can be 95% confident that the cheaper material will cool down from 0.1 to 2 minutes faster than the standard material. Depending on the difference in cost, it may be worthwhile to switch to the cheaper material but increase the amount used and obtain comparable cool down times to the standard material. How much extra material would be needed is not addressed by this data and further study is suggested.

2. Data is collected on the amount of time spent by students at the ISU Recreation Facility. Thirty students, 15 males and 15 females are observed. The elapsed time (minutes) spent during one visit to the area that has the aerobic fitness devices, weight machines and free weights is recorded for each student.

(a) Mann-Whitney-Wilcoxon test.

Males, X	Rank X	Females, Y	Rank Y
41	3.0	32	1.0
52	8.0	39	2.0
53	9.5	45	4.0
62	14.0	49	5.0
65	16.5	49	6.0
66	18.0	51	7.0
68	20.0	53	9.5
69	21.0	54	11.0
72	22.0	56	12.5
74	24.0	56	12.5
75	25.0	63	15.0
77	26.0	65	16.5
84	27.0	67	19.0
87	29.0	73	23.0
93	30.0	86	28.0
	$W_X = 293.0$		$W_Y = 172.0$

H: $\eta_X = \eta_Y$
A: $\eta_X > \eta_Y$

$U_X = 293 - \frac{15(16)}{2} = 173 \quad U_Y = 172 - \frac{15(16)}{2} = 52$

mean of U $= \frac{mn}{2} = \frac{15(15)}{2} = 112.5$

std dev of U $= \sqrt{\frac{mn(m+n+1)}{12}} = \sqrt{\frac{15(15)(31)}{12}} = \sqrt{581.25} = 24.11$

P-value $= 2Pr(U_X \geq 173) \doteq 2Pr\left(Z \geq \frac{173-0.5-112.5}{24.11}\right) = 2Pr(Z \geq 2.49) = 2(0.0064) = 0.0128$

Since the P-value is so small we should Reject H. The median time spent at the ISU Recreational Facility for males is significantly longer than that for females.

(b) Mann-Whitney-Wilcoxon test.

Males, X	Rank X	Females, Y	Rank Y
41	4.0	32	1.0
52	9.0	36	2.0
53	10.5	39	3.0
62	15.0	45	5.0
65	17.5	49	6.0
66	19.0	49	7.0
68	21.0	51	8.0
69	22.0	53	10.5
72	23.0	54	12.0
74	25.0	56	13.5
75	26.0	56	13.5
77	27.0	63	16.0
84	28.0	65	17.5
87	29.0	67	20.0
93	30.0	73	24.0
	$W_X = 306.0$		$W_Y = 159.0$

H: $\eta_X = \eta_Y$
A: $\eta_X > \eta_Y$

$U_X = 306 - \frac{15(16)}{2} = 186 \quad U_Y = 159 - \frac{15(16)}{2} = 39$

mean of U $= \frac{mn}{2} = \frac{15(15)}{2} = 112.5$

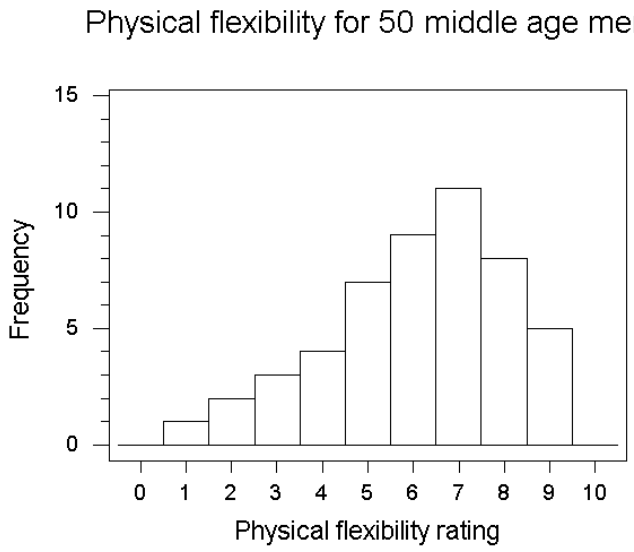
std dev of U $= \sqrt{\frac{mn(m+n+1)}{12}} = \sqrt{\frac{15(15)(31)}{12}} = \sqrt{581.25} = 24.11$

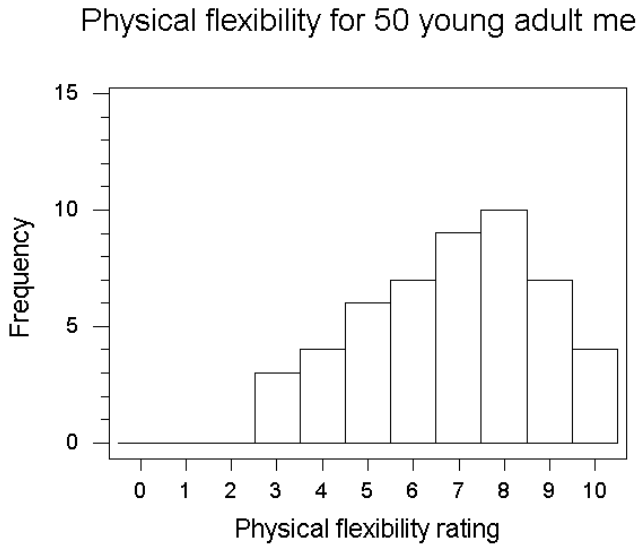
P-value $= 2Pr(U_X \geq 186) \doteq 2Pr\left(Z \geq \frac{186-0.5-112.5}{24.11}\right) = 2Pr(Z \geq 3.03) = 2(0.0012) = 0.0024$

Since the P-value is so small we should Reject H. The median time spent at the ISU Recreational Facility for males is significantly longer than that for females.

3. Fifty Middle-Aged Men and fifty Young Adult Men were rated on physical flexibility. The scale went from 1, least flexible, to 10, most flexible. The table below (see part (d)) shows a summary of these ratings. For example, no young adult men had a flexibility rating of 1, while 1 middle-aged man received this rating.

   (a) Histograms of physical flexibility scores.





   (b) The histograms indicate left skewed distributions. An assumption that physical flexibility scores are normally distributed is not reasonable. However, the sample sizes are quite large (50 in each group) so that the use of a two-sample t-test could be reasonable.

   (c) Tukey's quick (pocket) test statistic is 7. This is significant at the 2.5% level. It appears that young adult men are more physically flexible than middle aged men.

(d) Mann-Whitney-Wilcoxon test.

t_j	Ranks	Physical Flexibility Rating	Young Adult Men, X	Rank X	W_X	Middle Aged Men, Y	Rank Y	W_Y
1	1	1	0			1	1.0	1.0
2	2- 3	2	0			2	2.5	5.0
6	4- 9	3	3	6.5	19.5	3	6.5	19.5
8	10- 17	4	4	13.5	54.0	4	13.5	54.0
13	18- 30	5	6	24.0	144.0	7	24.0	168.0
16	31- 46	6	7	38.5	269.5	9	38.5	346.5
20	47- 66	7	9	56.5	508.5	11	56.5	621.5
18	67- 84	8	10	75.5	755.0	8	75.5	604.0
12	85- 96	9	7	90.5	633.5	5	90.5	452.5
4	97-100	10	4	98.5	294.0	0		
					2778.0			2272.0

H: $\eta_X = \eta_Y$
A: $\eta_X > \eta_Y$

$U_X = 2778 - \frac{50(51)}{2} = 1503 \quad U_Y = 2272 - \frac{50(51)}{2} = 997$

mean of U $= \frac{mn}{2} = \frac{50(50)}{2} = 1250$

std dev of U $= \sqrt{\frac{mn}{12}\left[(m+n+1) - \frac{\sum t_j(t_j^2-1)}{(m+n)(m+n-1)}\right]}$

std dev of U $= \sqrt{\frac{50(50)}{12}\left[101 - \frac{2(3)+6(35)+8(63)+13(168)+16(255)+20(399)+18(323)+12(143)+4(15)}{100(99)}\right]}$

$= \sqrt{\frac{2500}{12}\left[101 - \frac{22554}{9900}\right]} = \sqrt{20567.05} = 143.41$

P-value $= Pr(U_X \geq 1503) \doteq Pr\left(Z \geq \frac{1503-0.5-1250}{143.41}\right) = Pr(Z \geq 1.76) = 0.0392$

Since the P-value is small (less than 0.05) we should Reject H. The median physical flexibility score of young adult men is significantly greater than that of middle age men.