Stat 403 - Solution to Assignment 1

Turned in Thursday, September 7, 2000

3.28 Let θ be the probability that a woman juror is selected. In order to see if the number of women selected for a jury is consistent with the 65hypotheses:

$$\begin{aligned} H : \theta &= 0.65 \\ A : \theta &< 0.65 \end{aligned}$$

There are 3 successes (women jurors) and 6 failures (men jurors). The P-value can be found in several way from Table E in the book.

$$\begin{aligned} P - value &= Pr(Bin(9, 0.65) \leq 3) = 0.0536 \\ P - value &= Pr(Bin(9, 0.35) \geq 6) = 1 - 0.9464 = 0.0536 \end{aligned}$$

Although this is not significant at the 5% level, it is really too close to call. With such a small sample size it is difficult to establish discrimination on the basis of sex.

3.31 Let θ be the proportion of people using drugs after recent changes in the drug laws on drug use among teenagers. In order to see if the proportion of people using drugs has increased from the 7% that used before the laws changed we test the hypotheses:

$$\begin{aligned} H : \theta &= 0.07 \\ A : \theta &> 0.07 \end{aligned}$$

There are 78 out of 1000 people who currently use drugs. Since n=1000 is so large, we use a normal approximation to the binomial (with a correction for continuity) to compute an approximate P-value.

$$\begin{aligned} P - value &= Pr(Bin(1000, 0.07) \geq 78) \\ &\doteq Pr\left(Z \geq \frac{78 - 0.5 - 1000(0.07)}{\sqrt{1000(0.07)(0.93)}}\right) \\ &= Pr\left(Z \geq \frac{77.5 - 70}{\sqrt{65.10}}\right) \\ &= Pr\left(Z \geq \frac{7.5}{8.07}\right) \\ &= Pr(Z \geq 0.93) = 0.1762 \end{aligned}$$

Since the P-value is not small, we fail to reject (accept) the null hypothesis. There is not sufficient evidence to indicate that the proportion of people using drugs has increased since the new laws were passed.

3.32 Let θ be the proportion of people with colds given the new drug who recover within 2 weeks. In order to show that the new drug is effective, this proportion should be higher than what we commonly see, 70%. We test the hypotheses:

$$\begin{aligned} H : \theta &= 0.70 \\ A : \theta &> 0.70 \end{aligned}$$

Only 16 of 20 cold suffers treated with the drug recover within 2 weeks. The P-value using Table E is:

$$\begin{aligned} P - value &= Pr(Bin(20, 0.70) \geq 16) \\ &= 1 - Pr(Bin(20, 0.70) \leq 15) \\ &= 1 - 0.7625 = 0.2375 \end{aligned}$$

The approximate P-value using a normal approximation with a correction for continuity is:

$$\begin{aligned} P - value &= Pr(Bin(20, 0.70) \geq 16) \\ &\doteq Pr\left(Z \geq \frac{16 - 0.5 - 20(0.70)}{\sqrt{20(0.70)(0.30)}}\right) \\ &= Pr\left(Z \geq \frac{15.5 - 14}{\sqrt{4.2}}\right) \\ &= Pr\left(Z \geq \frac{1.5}{2.05}\right) \\ &= Pr(Z \geq 0.73) = 0.2327 \end{aligned}$$

The P-value, both exact and approximate, is not small. We cannot reject the null hypothesis. There is not sufficient evidence to conclude that the new drug is effective in curing the common cold.

3.1 A television tube is of adequate quality if the median lifetime is 500 hours. If the median lifetime is less than 500 hours then there is trouble. If the median lifetime is greater than 500 hours the tubes are better than the competitors. Twenty five tubes from Supplier B are tested until 500 hours and 22 survive, only 3 fail before 500 hours. If the true median is 500 hours then the probability of survival at 500 hours should be the same as the probability of failure at 500 hours (at 500 hours we should see about half of the tubes failing). The statement of the problem is a bit confusing since it suggests that one guard against Supplier B's tubes being inferior. This would suggest hypotheses:

$$\begin{aligned} H : \eta = 500 \quad or \quad p_+ = p_- \\ A : \eta < 500 \quad or \quad p_+ < p_- \end{aligned}$$

The approximate P-value for these hypotheses would be:

$$\begin{aligned} P-value &= Pr(Bin(25,0.50) \leq 22) \\ &\doteq Pr\left(Z \leq \frac{22+0.5-25(0.50)}{\sqrt{25(0.50)(0.50)}}\right) \\ &= Pr\left(Z \leq \frac{22.5-12.5}{\sqrt{6.25}}\right) \\ &= Pr\left(Z \leq \frac{10}{2.5}\right) \\ &= Pr(Z \leq 4.0) > 0.9998 \end{aligned}$$

Clearly, there is no evidence that the tubes from Supplier B are inferior.

If one were to test if Supplier B's tubes were superior, then the hypotheses would be:

$$\begin{aligned} H &: \eta = 500 \quad or \quad p_+ = p_- \\ A &: \eta > 500 \quad or \quad p_+ > p_- \end{aligned}$$

The approximate P-value for the sign test would be:

$$\begin{aligned} P-value &= Pr(Bin(25,0.50) \geq 22) \\ &\doteq Pr\left(Z \geq \frac{22-0.5-25(0.50)}{\sqrt{25(0.50)(0.50)}}\right) \\ &= Pr\left(Z \geq \frac{21.5-12.5}{\sqrt{6.25}}\right) \\ &= Pr\left(Z \geq \frac{9}{2.5}\right) \\ &= Pr(Z \geq 3.6) < 0.0002 \end{aligned}$$

This P-value leads us to reject the null hypothesis in favor of the alternative that says that Supplier B's tubes are actually superior in that they have a median lifetime longer than 500 hours.

3.3 A success is a yes answer to the question about the value of the course. Reasonable hypotheses would be:

$$\begin{aligned} H &: p_+ = p_- \\ A &: p_+ > p_- \end{aligned}$$

Of 32 returned questionnaires, 25 answered yes. The approximate P-value for the sign test would be:

$$P-value = Pr(Bin(32,0.50) \geq 25)$$

$$
\begin{aligned}
&\doteq Pr\left(Z \geq \frac{25 - 0.5 - 32(0.50)}{\sqrt{32(0.50)(0.50)}}\right) \\
&= Pr\left(Z \geq \frac{24.5 - 16}{\sqrt{8}}\right) \\
&= Pr\left(Z \geq \frac{8.5}{2.83}\right) \\
&= Pr(Z \geq 3.01) = 0.0013
\end{aligned}
$$

The P-value is so small that it is unlikely that yes and no responses are equally likely. It is reasonable to say that of the people who responded, there are more positive reactions to the course than negative. However, nonresponse is a problem. There are 17 people who did not respond. If they all would have answered no then the results would have been quite different. The P-value would have been about 0.5, indicating positive and negative reactions were equally likely. As a matter of fact, if as few as 11 of those nonrespondents would have answered no, then we would conclude that positive and negative reactions could be equally likely.

3.5 A success is when the given brand is considered superior. The given brand is considered superior in 38 or the 58 trials where a preference is given. The hypotheses are:

$$
\begin{aligned}
H &: p_+ = p_- \\
A &: p_+ > p_-
\end{aligned}
$$

The approximate P-value for the sign test would be:

$$
\begin{aligned}
P - value &= Pr(Bin(58, 0.50) \geq 38) \\
&\doteq Pr\left(Z \geq \frac{38 - 0.5 - 58(0.50)}{\sqrt{58(0.50)(0.50)}}\right) \\
&= Pr\left(Z \geq \frac{37.5 - 29}{\sqrt{14.5}}\right) \\
&= Pr\left(Z \geq \frac{8.5}{3.81}\right) \\
&= Pr(Z \geq 2.23) = 0.0129
\end{aligned}
$$

This is a statistically significant result. It is not likely that 38, or more, people would prefer the given brand if the two brands had equal probabilities of being chosen. Again, the 6 people who had no preference is a bit bothersome. If all 6 actually had a preference for the competing brand, the P-value would be 0.0838 and one would say that the two brands could be equal in terms of preference.