

## STATISTICS 402 – Assignment 9 – Solution

1. You are asked to help design an experiment involving the effect of exam aids on the performance of students on the second exam in an introductory statistics course. The four treatments are (1) no exam aids, (2) calculator only, (3) formula sheet only and (4) calculator and formula sheet. An additional factor (individual versus group work in recitation) is also of interest. For this experiment, 200 students in a large lecture introductory statistics course have agreed to participate. The students are divided, at random, into 10 recitation sections of 20 students each. Half of the recitation sections, selected at random, will do individual work on problems similar to the ones that will appear on the exam. The other half of the recitation sections will work in small groups on problems similar to the ones that will appear on the exam. Each recitation section will be divided, at random, into 4 groups of 5 students for the exam. Within each recitation section the groups will be randomly assigned to the four treatments: (1) no exam aids, (2) calculator only, (3) formula sheet only and (4) calculator and formula sheet for the exam. Exam score will be the response.
  - a. Explain why this is a split plot/repeated measures design. In this explanation be sure to address the following:
    - i. What are the “whole plots”?
    - ii. What is the “whole plot,” between subjects, factor?
    - iii. What are the “sub plots”?
    - iv. What is the “sub plot,” within subjects factor?

**This is a split plot design because there are experimental units of two different sizes, recitation sections and groups of 5 students within each recitation section. The whole plots are the recitation sections. The whole plot factor is Individual/Group Work. The sub plots are the groups of students within each recitation section. The sub plot factors are Calculator (Yes/No) and Formula Sheet (Yes/No).**

- b. Construct a partial ANOVA table indicating sources of variation and degrees of freedom. Also indicate how to construct the appropriate F tests for determining the statistical significance of the model effects.

Source	df	MS	F
<b>Individual/Group Work</b>	<b>1</b>	$MS_{\text{Work}}$	$MS_{\text{Work}}/MS_{\text{Recitation[Work]}}$
<b>Recitation[Work]</b>	<b>8</b>	$MS_{\text{Recitation[Work]}}$	
<b>Calculator</b>	<b>1</b>	$MS_{\text{Calc}}$	$MS_{\text{Calc}}/MS_{\text{Error}}$
<b>Formula Sheet</b>	<b>1</b>	$MS_{\text{Form}}$	$MS_{\text{Form}}/MS_{\text{Error}}$
<b>Calc*Form</b>	<b>1</b>	$MS_{\text{Calc*Form}}$	$MS_{\text{Calc*Form}}/MS_{\text{Error}}$
<b>Work*Calc</b>	<b>1</b>	$MS_{\text{Work*Calc}}$	$MS_{\text{Work*Calc}}/MS_{\text{Error}}$
<b>Work*Form</b>	<b>1</b>	$MS_{\text{Work*Form}}$	$MS_{\text{Work*Form}}/MS_{\text{Error}}$
<b>Work*Calc*Form</b>	<b>1</b>	$MS_{\text{Work*Calc*Form}}$	$MS_{\text{Work*Calc*Form}}/MS_{\text{Error}}$
<b>Error</b>	<b>184</b>	$MS_{\text{Error}}$	
<b>C. Total</b>	<b>199</b>		

- c. In this experiment, we have not accounted for individual differences in students' aptitude for statistics, study and exam taking skills. Where does this variability in students end up in the ANOVA table? Explain briefly.

**The variation in students ends up in the Error term. Therefore, looking for sub plot factor effects may be more difficult. However, we also have a large number of degrees of freedom for this Error term. Some of this variation does end up in the Recitation[Work] (whole plot error) term. However, the pluses and minuses tend to average out when we take Recitation[Work] averages.**

2. In order to increase the strength, refine the grain and homogenize the structure of steel, steel is heated above a critical temperature, quenched in water and then air-cooled. An experiment is performed to determine the effect of temperature and heat treatment time on the strength of steel. Two temperatures and three times are selected. The experiment is performed by heating the oven to a randomly selected temperature and inserting 3 specimens. After 10 minutes one specimen is chosen at random and removed, after another 10 minutes a second specimen is chosen at random and removed, and after another 10 minutes the third specimen is removed. Then the temperature is reset and the process is repeated. Each temperature is replicated 4 times in a completely random order.

Below are the data arising from this split plot (repeated measures) design. The run gives the randomized order of the temperature settings.

	Temperature				Temperature		
	1500° F				1600° F		
	Time				Time		
Run	10 min	20 min	30 min	Run	10 min	20 min	30 min
3	52	56	61	7	89	91	62
8	50	52	59	2	72	80	69
1	58	64	71	4	73	81	69
6	48	54	59	5	88	92	64

- a. What is the response?

**The response is the strength of the steel.**

- b. What are the experimental units?

**The experimental units are the specimens.**

- c. What is the whole plot, between subjects, factor?

**The whole plot, between subjects, factor is the temperature of the oven.**

- d. What is the sub plot, within subjects, factor?

**The sub plot, within subjects, factor is the time.**

- e. Use the computer program of your choice to analyze these data keeping in mind that this is a split plot (repeated measures) design. Be sure to include plots of main effects and an interaction plot as well as an analysis of the distribution of residuals. Turn in the computer output with your assignment.

**See the JMP output for this problem.**

- f. Are the two temperatures significantly different in terms of mean strength? Report the appropriate F- and P- values to support your answer.

**Yes. F = 40.06, P-value = 0.0007. The small P-value indicates that the difference in temperature means is statistically significant.**

- g. Are there significant differences among the times? Report the appropriate F- and P-values to support your answer.

**Yes. F = 4.50, P-value = 0.0348. The P-value is less than 0.05 indicating that there are some differences among the time means.**

- h. Compute the LSD for comparing mean strengths for the three times. What, if any, times have statistically significant differences?

$$\text{LSD} = 2.17881 * 2.4037 = 5.24$$

Level	Mean	
20 minutes	71.25	A
10 minutes	66.25	A B
30 minutes	64.25	B

**The only statistically significant difference in mean strength is between the 20 minute and 30 minute times.**

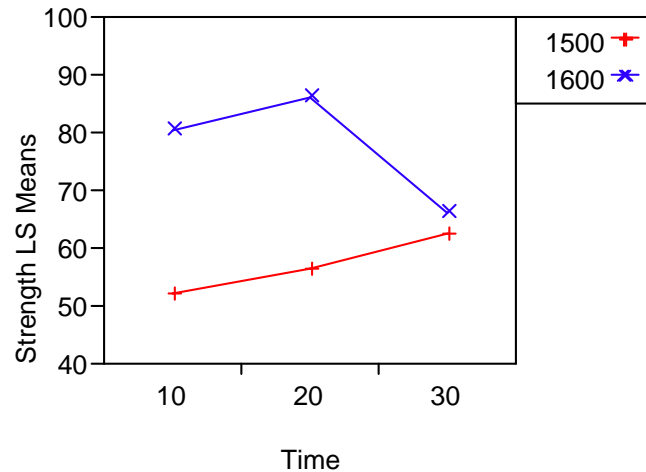
- i. Is there a statistically significant linear relationship between time and strength? Support your answer with an appropriate linear contrast and test of hypothesis.

$$SS_{\text{Linear}} = \frac{8(-1 * 66.25 + 0 * 71.25 + 1 * 64.25)^2}{(1 + 0 + 1)} = \frac{32}{2} = 16$$

$$F = \frac{16}{23.11} = 0.69 \quad P - \text{value} = 0.4216$$

No. The P-value is greater than 0.05 indicating that the linear contrast (and thus the linear relationship between time and strength) is not statistically significant.

- j. Comment on the interaction plot. Be sure to tell me what you see (be specific) and what this means.



Going from 10 to 20 minutes, the lines are virtually parallel (indicating no interaction between time and temperature). Going from 20 to 30 minutes, the lines are quite different (for the 1500° F temperature the mean strength increases while for the 1600 ° F temperature the mean strength drops quite a bit). Therefore, the relationship between time and strength is different for the two temperatures. This is an indication of an interaction effect.

- k. Is there a statistically significant interaction between time and temperature? Report the appropriate F- and P-values to support your answer.

**Yes.  $F = 18.78$ ,  $P\text{-value} = 0.0002$ . The small P-value indicates that the interaction between time and temperature is statistically significant.**

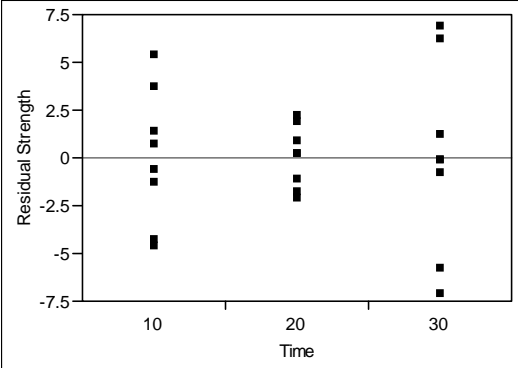
- l. Comment on the distribution of residuals. Tell me what you see in the various plots (Normal quantile plot, box plot and histogram) and indicate what this tells you about the condition of normally distributed errors.

**Because there are two error terms (Run[Temperature] and Error) there are two sets of residuals. There are only 8 Run[Temperature] residuals.**

1500°	1600°
-0.6667	3.1667
-3.3333	-3.8333
7.3333	-3.1667
-3.3333	3.8333

The residuals for 1500° are more variable than those for 1600°, but it is probably not enough to cause concern. It is difficult to say much about the Normal distribution condition with only 8 residuals.

For the subplot factor (Time) there are more residuals to look at.



For Time = 20 there is less variability than times 10 or 30. Time = 30 has a standard deviation that is more than 3 times that of Time = 20. The condition of equal standard deviations is not met.

The normal quantile plot seems to have three groups, four negative residuals above the diagonal line, a bunch of residuals increasing from below the line to above the line, and 4 positive residuals below the line. The box plot is symmetric with one potential outlier (to the left). The histogram is mounded in the middle and fairly symmetric. Although not perfect, the residuals could have come from a normal distribution so the condition of normally distributed errors could be considered met.

