

STATISTICS 402 - Assignment 6 Solution

1. An engineer wishes to design a battery that will function for a long time in a variety of temperatures. The engineer has three different plate materials to use in building the battery. Once the battery goes into production and is shipped to customers, the engineer has no control over the temperatures that the battery may be exposed to. However, the engineer can manipulate the temperature in the laboratory for testing the lifetime of prototype batteries. To test the lifetime, a battery is placed in a chamber where the temperature is maintained at a specified level. The battery is hooked to a circuit that has a voltage meter and a timer. When the voltage drops below a specified threshold the timer stops giving you the lifetime of the battery in hours. The engineer builds prototype batteries with each of the three types of plate material. All other components of the batteries e.g. casings, terminals, design, etc. are the same. Batteries are tested at three temperature levels: -10 C , 20 C , and 50 C . Several batteries will be tested at each combination of temperature and plate material. The testing of the batteries will be done in a completely randomized order.

- a) Identify the response, conditions and experimental units.

Response: Lifetime of batteries (hours).

Conditions: Combinations of Plate Material and Temperature.

Experimental Units: Batteries

- b) Give an example of an outside variable that is controlled in the study. How is it controlled?

The components of the batteries e.g. casings, terminals, design, etc. are all the same.

- c) How many treatment combinations are there? List all the treatment combinations.

There are 9 treatment combinations: Material A at -10 C , Material A at 20 C , Material A at 50 C , Material B at -10 C , Material B at 20 C , Material B at 50 C , Material C at -10 C , Material C at 20 C , Material C at 50 C .

- d) If the experimenter wishes to detect a difference in means for the three different plate materials as small as 1.0 standard deviations with $\text{Alpha}=0.05$ and $\text{Beta}=0.10$, how many batteries are needed?

With three different plate material you will need 27 batteries for each plate material for a total of 81 batteries.

- e) With the number of batteries you found in d), how small a difference in means for the treatment combinations of plate material and temperature can be found with Alpha=0.05 and Beta=0.10?

There are 9 treatment combinations therefore there would be 9 batteries for each treatment combination. You would be able to detect a difference in treatment means of between 2 and 2.5 standard deviations.

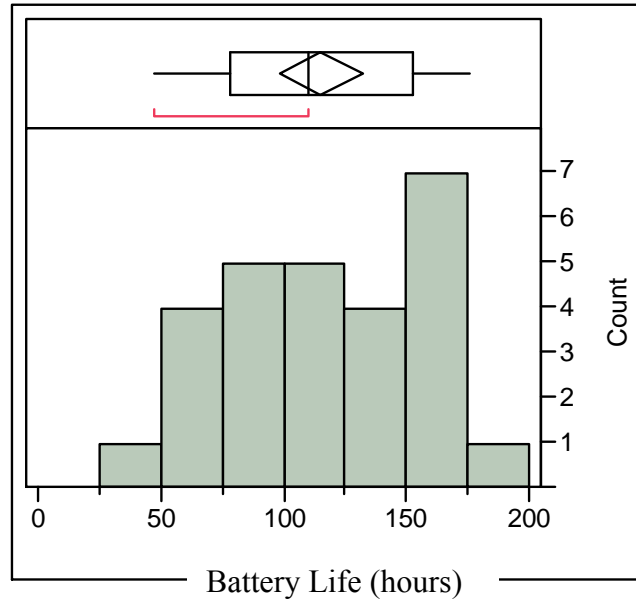
- f) The engineer comes to you for advice on randomization. She explains she can only afford to build and test 3 batteries for each combination of plate material and temperature. Explain to the experimenter how to use complete randomization to build and test the batteries. Do the randomization and turn in the randomized order.

One randomization could be used for both building and testing the batteries. One needs to randomly order the 27 runs. To use JMP put the 3 replications of the 9 treatment combinations in two columns labeled Plate Material and Temperature. In a third column, labeled Random Order, use Cols – Formula – Random – Column Shuffle to randomly shuffle the integers from 1 to 27. You can then sort the JMP table according to the Random Order column, then the first row will be the first battery built and tested, the second row the second battery built and tested, etc.

Plate	Temp	Order	Plate	Temp	Order	Plate	Temp	Order
B	-10	1	A	50	10	A	20	19
A	-10	2	A	50	11	B	50	20
B	50	3	C	50	12	A	50	21
C	-10	4	A	20	13	C	20	22
B	-10	5	B	20	14	C	-10	23
A	-10	6	C	50	15	A	-10	24
C	20	7	C	50	16	B	20	25
A	20	8	B	20	17	C	20	26
C	-10	9	B	-10	18	B	50	27

2. The experiment described in problem 1 is run. The data are available on the course web site in the file BatteryLife.JMP. Download this file and use JMP to help with the analysis of the data.
- a) Analyze and comment on the distribution of battery life.

The distribution of battery life appears to have more than one mode (one between 75 to 125 hours, and another between 150 and 175 hours). The average battery life for the sample of 27 batteries is 115.3 hours. The standard deviation of battery life is 42.64 hours.



100.0%	maximum	176.0	Mean	115.25926
75.0%	quartile	153.0	Std Dev	42.638546
50.0%	median	110.0	N	27
25.0%	quartile	78.0		
0.0%	minimum	47.0		

- b) Fit a model that includes Plate Material, Temperature and Plate*Temp interaction effects. Give the value of R^2 and the value of the estimate of the error standard deviation, σ .

$R^2 = 0.942$, so the full factorial model explains 94.2% of the variation in battery life.

RMSE = 12.31 years is the estimate of the error standard deviation.

- c) Test the hypothesis that there are no differences among the population treatment means. Be sure to include the F statistic, P-value, decision, reason for the decision and a conclusion.

$F = 36.74$, $P\text{-value} < 0.0001$. Because the P-value is so small we should reject the null hypothesis of no differences among treatment population means. Therefore there are some treatment combinations of plate material and temperature that do result in different mean lifetimes for batteries.

- d) Test the hypothesis that the plate material effects are all zero. Be sure to include the F statistic, P-value, decision, reason for the decision and a conclusion.

F = 12.31, P-value = 0.0004. Because the P-value is so small we should reject the null hypothesis of no plate material effects. Therefore there are some plate materials that affect the mean lifetimes for batteries.

- e) Where are the statistically significant differences among the plate material means? Use the LSD (Student's t) multiple comparisons procedure.

LSD = 2.10092(5.80336) = 12.19. Any difference in plate material means greater than 12.19 would be considered statistically significant.

Level		Least Sq Mean	Comparison	Difference in Means
C	A	129.88889	C - B	15.11 > 12.19
B	B	114.77778	C - A	28.78 > 12.19
A	C	101.11111	B - A	13.67 > 12.19

Levels not connected by same letter are significantly different.

- f) Test the hypothesis that the temperature effects are all zero. Be sure to include the F statistic, P-value, decision, reason for the decision and a conclusion.

F = 130.35, P-value < 0.0001. Because the P-value is so small we should reject the null hypothesis of no temperature effects. Therefore there are some temperatures that affect the mean lifetimes for batteries.

- g) Where are the statistically significant differences among the means for the different temperatures? Use the HSD (Tukey-Kramer) multiple comparisons procedure.

HSD = 2.55216(5.80336) = 14.81. Any difference in plate material means greater than 14.81 would be considered statistically significant.

Level		Least Sq Mean	Comparison	Difference in Means
-10	A	163.55556	-10 to 20	51.33 > 14.81
20	B	112.22222	-10 to 50	93.56 > 14.81
50	C	70.00000	20 to 50	42.22 > 12.19

Levels not connected by same letter are significantly different.

- h) Is a linear contrast on temperature statistically significant? If it is, what does this indicate?

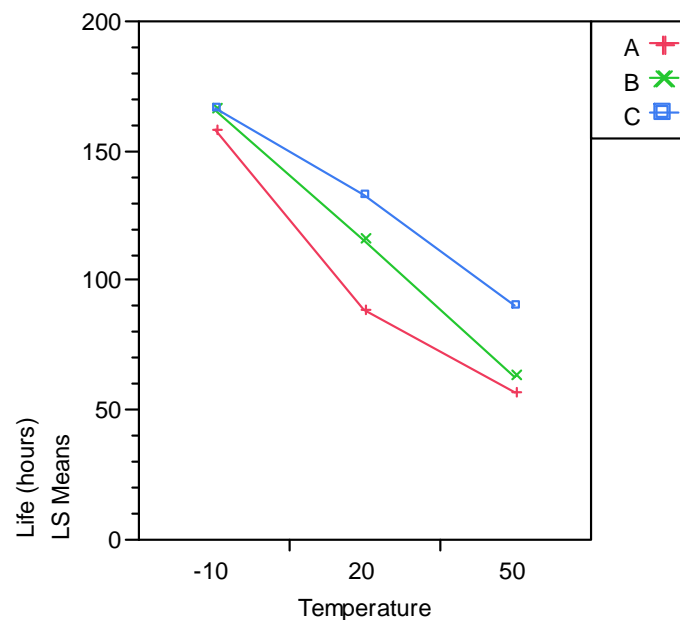
SS_{Linear} = 39386.89, F = 259.88, P-value < 0.0001. Because the P-value is so small, the linear contrast in temperature is statistically significant.

This indicates that the relationship between battery life and temperature has a statistically significant linear component.

- i) Test the hypothesis that the interaction effects are all zero. Be sure to include the F statistic, P-value, decision, reason for the decision and a conclusion.

F = 2.14, P-value = 0.1171. Because the P-value is not small we cannot reject the null hypothesis of no interaction effects. Therefore there is not a statistically significant interaction between plate material and temperature.

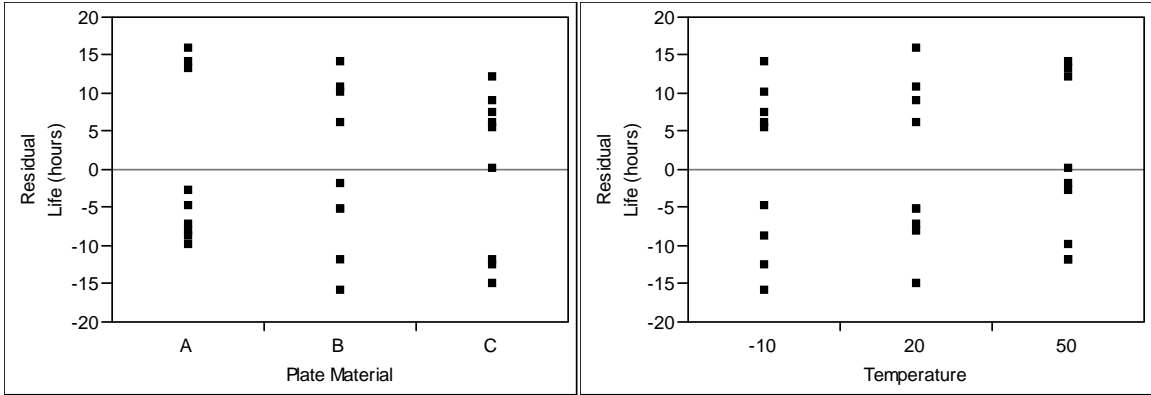
- j) Construct and comment on the interaction plot.



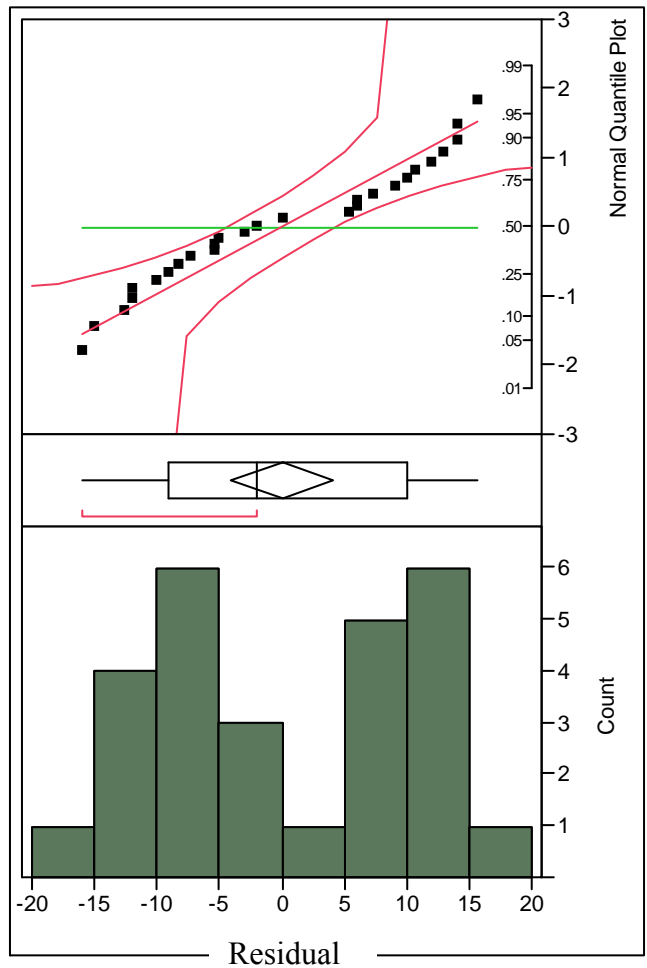
For all three plate materials, as temperature increases, average battery life tends to decrease. Although the lines connecting the means are not exactly parallel, they are fairly close to being parallel. This means that the effect of temperature on lifetime of batteries is the same for all three materials. This is consistent with the test of hypothesis that says that there is not a statistically significant interaction between plate material and temperature.

- k) Construct plots of residuals versus plate material and residuals versus temperature in the experiment. Comment on the plots.

The spreads of the residuals are approximately the same for all plate materials and all temperatures. This supports the condition of equal standard deviations.



- 1) Analyze the distribution of residuals. Comment on what the analysis of residuals says about the conditions necessary for statistical analysis.



The histogram has two mounds, one between -10 and -5 and the other between 10 and 15. There could be two normal distributions for the residuals rather than one. The Normal quantile plot also shows two groups with most of the negative residuals above the diagonal line and most of the positive residuals below the diagonal line. The normal distribution condition is not met.