Solution to Assignment 2

- **Convenience Assignment**

It is easiest to plant one variety on 18 plots on one side of the field and the other variety on the 18 plots on the other side. On the back is a picture of such and assignment and the yields, in bushels per acre, for each plot. The yields and a summary of those yields are given below.

A 130	A 149	A 139	B 155	B 137	B 145
A 149	A 133	A 152	B 131	B 147	B 136
A 141	A 156	A 137	B 146	B 132	B 148
A 150	A 142	A 155	B 136	B 152	B 133
A 139	A 155	A 139	B 147	B 137	B 153
A 155	A 138	A 150	B 137	B 145	B 136

Variety	n	mean	std. dev
A	18	144.9	8.29
B	18	141.8	7.65

$$s_p = \sqrt{\frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{(n_A + n_B - 2)}} = \sqrt{\frac{17(8.29)^2 + 17(7.65)^2}{34}} = \sqrt{63.6233} = 7.976$$

$$\text{std. error of difference} = s_p\sqrt{\frac{1}{n_A} + \frac{1}{n_B}} = 7.976\sqrt{\frac{1}{18} + \frac{1}{18}} = 2.659$$

Using a two-sample t-test, the value of the test statistic is

$$t = \frac{(144.9 - 141.8)}{2.659} = 1.17$$

with a two-sided P-value of 0.25. This indicates that although variety A has a slightly higher mean yield than variety B, the two varieties are not statistically different.

This is probably not a fair comparison. One side of the field may be more fertile than the other or get more water than the other. There could be many lurking variables that could be affecting the yields.

- **Systematic Assignment**

Many people think that an alternating sequence is a random, or at least an unbiased, sequence. On the back is a picture of an alternating pattern, like a checkerboard, and the yields, in bushels per acre, for each plot. The yields and a summary of those yields are given below.

A 130	B 137	A 139	B 155	A 149	B 145
B 137	A 133	B 140	A 143	B 147	A 148
A 141	B 144	A 137	B 146	A 144	B 148
B 138	A 142	B 143	A 148	B 152	A 145
A 139	B 143	A 139	B 147	A 149	B 153
B 143	A 138	B 138	A 149	B 145	A 148

Variety	n	mean	std. dev
A	18	142.3	5.75
B	18	144.5	5.37

$$s_p = \sqrt{\frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{(n_A + n_B - 2)}} = \sqrt{\frac{17(5.75)^2 + 17(5.37)^2}{34}} = \sqrt{30.9497} = 5.563$$

$$\text{std. error of difference} = s_p\sqrt{\frac{1}{n_A} + \frac{1}{n_B}} = 5.563\sqrt{\frac{1}{18} + \frac{1}{18}} = 1.854$$

Using a two-sample t-test, the value of the test statistic is

$$t = \frac{(142.3 - 144.5)}{1.854} = -1.19$$

with a two-sided P-value of 0.24. This indicates that although variety B has a slightly higher mean yield than variety A, the two varieties are not statistically different.

- **Random Assignment**

Below is one possible random assignment of varieties to plots. This randomization was accomplished using a six-sided die. Rolling the die twice gives a row number and a column number thus identifying a unique plot. This was done until 18 plots were assigned variety A. The remaining plots were assigned variety B. Below are the yields and a summary of those yields.

A 130	A 149	A 139	A 167	B 137	A 157
A 149	B 121	B 140	B 131	A 159	B 136
A 141	A 156	A 137	A 158	A 144	A 160
A 150	B 130	B 143	B 148	B 152	B 133
B 127	A 155	B 127	A 159	A 149	B 153
B 143	B 126	B 138	A 149	B 145	B 136

Variety	n	mean	std. dev
A	18	150.4	9.49
B	18	136.3	8.74

$$s_p = \sqrt{\frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{(n_A + n_B - 2)}} = \sqrt{\frac{17(9.49)^2 + 17(8.74)^2}{34}} = \sqrt{83.22385} = 9.123$$

$$\text{std. error of difference} = s_p\sqrt{\frac{1}{n_A} + \frac{1}{n_B}} = 9.123\sqrt{\frac{1}{18} + \frac{1}{18}} = 3.041$$

Using a two-sample t-test, the value of the test statistic is

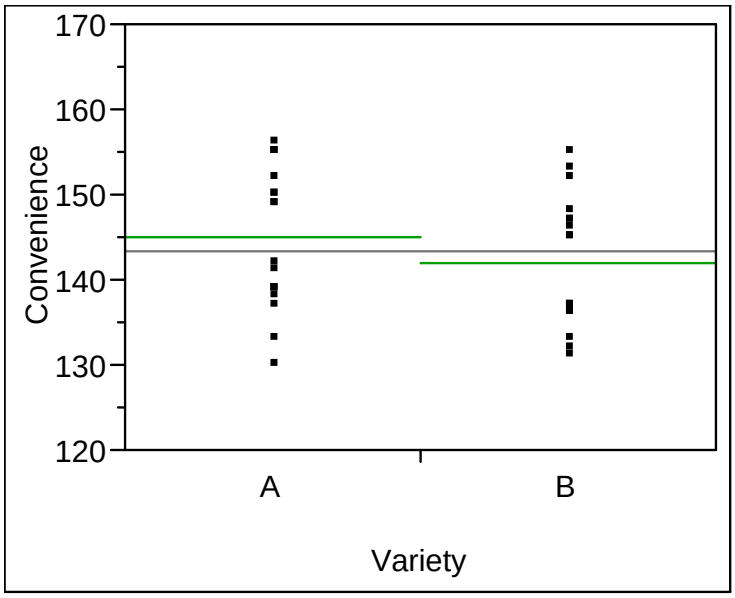
$$t = \frac{(150.4 - 136.3)}{3.041} = 4.64$$

A two-sample t-test on these data has a test statistic of 4.64 with a P-value of 0.0001. Clearly, there is a statistically significant difference between the mean yields for the two varieties. With this randomization variety A is over 14 bushels per acre better than variety B.

Closer examination of the “Truth” reveals that in each plot variety A is 12 bushels higher than variety B.

JMP Output

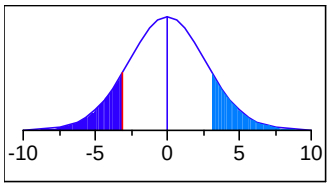
Yield using Convenience Assignment



t Test
B-A

Assuming equal variances

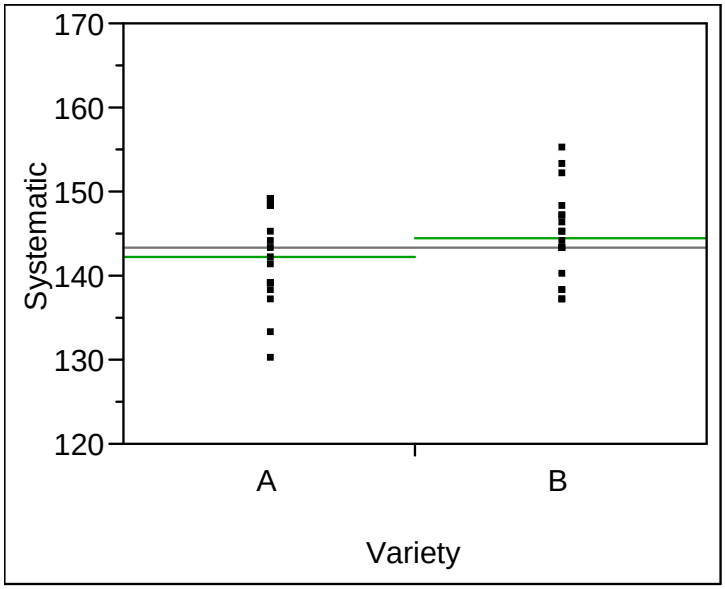
Difference	-3.1111	t Ratio	-1.17059
Std Err Dif	2.6577	DF	34
Upper CL Dif	2.2901	Prob > |t|	0.2499
Lower CL Dif	-8.5123	Prob > t	0.8750
Confidence	0.95	Prob < t	0.1250



Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean
A	18	144.944	8.28515	1.9528
B	18	141.833	7.64853	1.8028

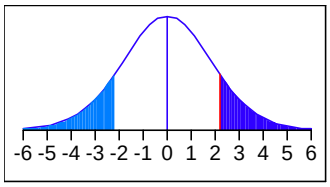
Yield using Systematic Assignment



t Test
B-A

Assuming equal variances

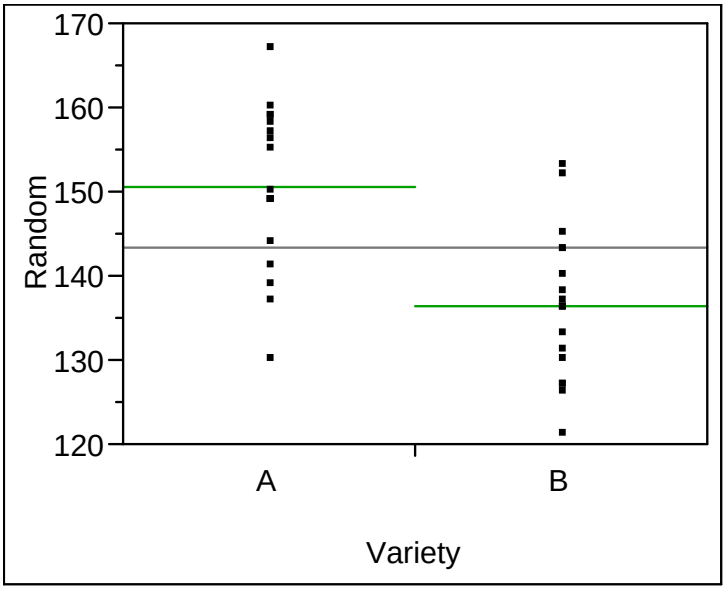
Difference	2.2222	t Ratio	1.198443
Std Err Dif	1.8543	DF	34
Upper CL Dif	5.9905	Prob > |t|	0.2390
Lower CL Dif	-1.5461	Prob > t	0.1195
Confidence	0.95	Prob < t	0.8805



Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean
A	18	142.278	5.74769	1.3547
B	18	144.500	5.37149	1.2661

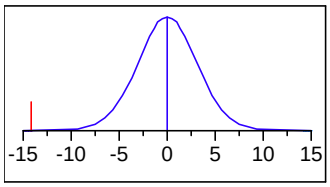
Yield using Random Assignment



t Test
B-A

Assuming equal variances

Difference	-14.111	t Ratio	-4.63811
Std Err Dif	3.042	DF	34
Upper CL Dif	-7.928	Prob > |t|	<.0001
Lower CL Dif	-20.294	Prob > t	1.0000
Confidence	0.95	Prob < t	<.0001



Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean
A	18	150.444	9.49441	2.2379
B	18	136.333	8.74475	2.0612