Statistics 402C Exam 1 Name:_____
February 23, 2000

INSTRUCTIONS: Read the questions carefully and completely. Answer the questions and show work in the space provided. This is the only work that I will look at. Partial credit cannot be given if work is not shown. Refer to the computer printout and graphs provided when appropriate. **Pace yourself**, do not spend too much time on any one problem. Point values for each problem are given.

1. **[20 pts]** A research psychologist wishes to investigate the difference in maze test times for mice trained using different levels of reinforcement; no reinforcement, positive reinforcement (food given for a correct choice), negative reinforcement (mild electric shock given for an incorrect choice). After training, each mouse will be run in a new maze and the time to complete the maze will be recorded.

   (a) **[4]** What are the response, conditions and units for this experiment?

   (b) **[3]** Natural variability in units is not desirable, what might the researcher do to try to reduce the natural variability in units?

   (c) **[3]** Thinking that variation in the response may be large, the researcher wishes to detect a difference in mean times as small as 0.8 standard deviations. With $\alpha = 0.05$ and $\beta = 0.05$, how many units does the researcher need to run the experiment as a completely randomized design?

(d) **[5]** The researcher can afford (unit cost, training time, etc.) to have only 27 units for the entire experiment. Give one choice of α, β and the size of a detectable difference that will meet this restriction. How does this choice compare to the choice in (c)?

(e) **[5]** Explain in some detail how the researcher can randomly assign units to conditions and randomize the order of the runs.

2. **[10 pts]** A friend, who knows you are taking Stat 402, comes to you with the following problem. The friend has run a designed experiment (CRD) with four treatments. When she looks at the SAS analysis the F test indicates NO SIGNIFICANT differences among the treatments. What is really strange is that if Treatment 4, which has the highest mean value, is not included in the analysis, the F test indicates the other 3 treatments ARE significantly different. What is going on? The SAS outputs for both analyses appear on the next page.

Class Level Information

Class	Levels	Values
TRMT	4	1 2 3 4

Number of observations in data set = 36

Dependent Variable: SCORE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	126.000000	42.000000	2.71	0.0615
Error	32	496.000000	15.500000		
Corrected Total	35	622.000000			

Source	DF	Anova SS	Mean Square	F Value	Pr > F
TRMT	3	126.000000	42.000000	2.71	0.0615

Level of TRMT	N	----------SCORE---------- Mean	SD
1	9	7.0000000	1.22474487
2	9	8.0000000	1.11803399
3	9	9.0000000	1.32287566
4	9	12.0000000	7.58287544

Class Level Information

Class	Levels	Values
TRMT	3	1 2 3

Number of observations in data set = 27

Dependent Variable: SCORE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	18.0000000	9.0000000	6.00	0.0077
Error	24	36.0000000	1.5000000		
Corrected Total	26	54.0000000			

Source	DF	Anova SS	Mean Square	F Value	Pr > F
TRMT	2	18.0000000	9.0000000	6.00	0.0077

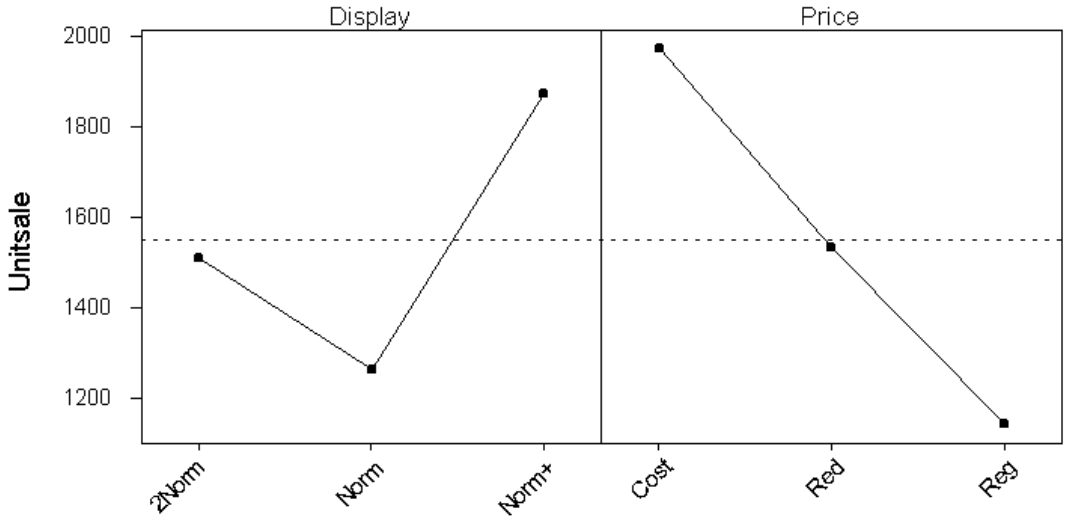
3. **[50 pts]** Most short-run supermarket strategies such as price reductions, media advertising, and in-store promotions and displays are designed to increase unit sales of particular products temporarily. Factorial designs have been employed to evaluate the effectiveness of such strategies. Two factors examined are Price Level (regular, reduced price and cost to the supermarket) and Display Level (normal display space, normal display space plus end-of-aisle display, twice the normal display space). A complete factorial experiment based on these two factors involves nine treatments. Suppose each treatment is applied three times to a particular product at a particular supermarket. Each application lasts a full week and the response variable of interest is unit sales for the week. To minimize treatment carryover effects, each treatment is preceded and followed by a week in which the product is priced at its regular price and is displayed in its normal manner. The data are given below.

		Price		
		Regular	Reduced	Cost
	Normal	989	1211	1577
		1025	1215	1559
		1030	1182	1598
Display	Normal Plus	1191	1860	2492
		1233	1910	2527
		1221	1926	2511
	Twice Normal	1226	1516	1801
		1202	1501	1833
		1180	1498	1852

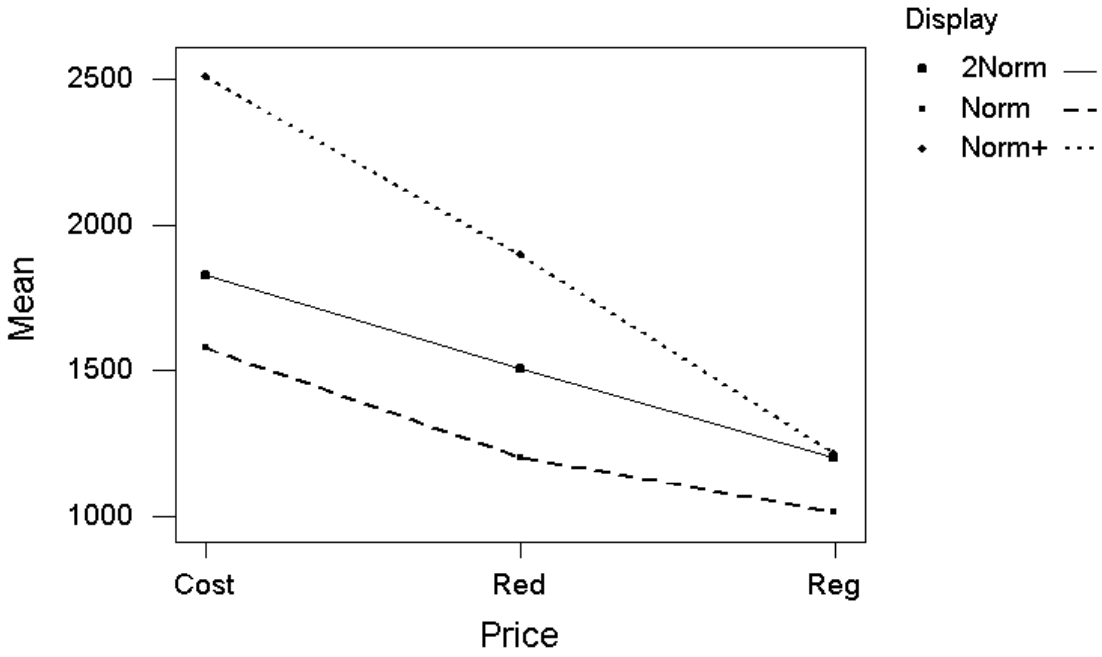
(a) **[5]** What is the response? What are the conditions? What are the units?

(b) **[10]** On the next page are main effects plots and an interaction plot. What do these plots tell you about the effects of Price? Display? and Price*Display interaction? Refer specifically to what you see in each plot and what that tells you about the effect.

Main Effects Plots of Display and Price



Interaction Plot for Display and Price



On the next page is output from SAS for the analysis of these data.

(c) **[5]** Are there significant differences in means for the three Price levels? Support your answer statistically.

(d) **[5]** Is there a significant interaction between Price and Display? Support your answer statistically.

(e) **[8]** Compute the value of the LSD for Display. Use 95% confidence. Which Display level means are significantly different?

(f) **[5]** The "linPrice" contrast in the SAS output is a linear contrast in the Price means. Is this contrast significant? Support your answer statistically. What does this contrast tell you about the relationship between Price and the response?

(g) **[6]** What combination of Price level and Display level would gives you the highest average response? The second highest?

(h) **[6]** For comparing all possible combinations of the 9 treatment means, the t table value that maintains an experimentwise error rate of 5% is $t^* = 3.7742$. Use this to compute an adjLSD. Are your two choices in (g) statistically different?

Class Level Information

Class	Levels	Values
DISPLAY	3	2Norm Norm Norm+
PRICE	3	Cost Red Reg

Number of observations in data set = 27

Dependent Variable: UNITSALE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	5291151.19	661393.90	1336.85	0.0001
Error	18	8905.33	494.74		
Corrected Total	26	5300056.52			

Source	DF	Type III SS	Mean Square	F Value	Pr > F
DISPLAY	2	1691392.52	845696.26	1709.37	0.0001
PRICE	2	3089053.85	1544526.93	3121.89	0.0001
DISPLAY*PRICE	4	510704.81	127676.20	258.07	0.0001

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
linPrice	1	3085956.06	3085956.06	6237.52	0.0001

Level of DISPLAY	N	UNITSALE Mean	UNITSALE SD
2Norm	9	1512.11111	271.710988
Norm	9	1265.11111	248.991689
Norm+	9	1874.55556	561.479544

Level of PRICE	N	UNITSALE Mean	UNITSALE SD
Cost	9	1972.22222	418.087544
Red	9	1535.44444	302.902670
Reg	9	1144.11111	99.136830

Level of DISPLAY	Level of PRICE	N	UNITSALE Mean	UNITSALE SD
2Norm	Cost	3	1828.66667	25.7746646
2Norm	Red	3	1505.00000	9.6436508
2Norm	Reg	3	1202.66667	23.0072452
Norm	Cost	3	1578.00000	19.5192213
Norm	Red	3	1202.66667	18.0092569
Norm	Reg	3	1014.66667	22.3681321
Norm+	Cost	3	2510.00000	17.5214155
Norm+	Red	3	1898.66667	34.4286702
Norm+	Reg	3	1215.00000	21.6333077

4. **[10 pts]** The three fundamental principles of a designed experiment are: control of outside variables, randomization and replications. Does the following scenario adhere to all three principles? If so, describe briefly how. If not, indicate how you would change the experiment.

   An experiment is to be performed on the effect of drill speed on the size of drilled holes. A single 0.25 inch drill bit will be used in a vertical head drilling machine. All drilling will be done by the same operator. A single bar of 6061 aluminum will be used. Four speeds will be investigated: 100, 150, 200 and 250 rpms. the operator will pick a speed at random from the four and then drill 20 holes at that speed. The process will be repeated; picking a drill speed at random form those remaining and drilling 20 holes until all 4 speed have been used. Each of the holes will be measured by the same technician using the same special caliper. The order of measurement will be random.

5. **[10 pts]** Another friend asks for your help with the design of his research project. The friend plans a series of 3 separate experiments. Each experiment will study the effect of a single treatment. You have convinced your friend that a control group is a good idea. The friend comes up with the following plan:

   - Experiment 1 : Treatment 1 vs. Control
   - Experiment 2 : Treatment 2 vs. Control
   - Experiment 3 : Treatment 3 vs. Control

   In each experiment the friend wants to be able to detect a difference in means of 3 standard deviations with $\alpha = 0.05$ and $\beta = 0.05$.

   (a) **[5]** How many experimental units are needed for each experiment? For all three experiments?

   (b) **[5]** Your friend has a total of 20 experimental units. Can your friend meet the requirements of detecting a 3 standard deviation difference in means with $\alpha = 0.05$ and $\beta = 0.05$? If so, how? Be specific.