Lecture 25: More on Block Designs

**Example**

*Response: torque on knee*
*Conditions: placement of feet*
  *Feet back, feet neutral, feet staggered*
*Experimental Material: men who have had knee replacement.

**Analysis of Variance**

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>2</td>
<td>91.60</td>
<td>45.8</td>
<td>31.84</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Subject</td>
<td>14</td>
<td>371.51</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>28</td>
<td>40.27</td>
<td>1.4383</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. Total</td>
<td>44</td>
<td>503.38</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Correct Analysis**

*The correct analysis matches the block design by including a source of variation due to the blocks (Subjects).*

**Incorrect Analysis**

*An incorrect analysis would ignore the fact that it was a block design and ignore blocks (Subjects) as a separate source of variation.*

**Analysis of Variance**

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>2</td>
<td>91.60</td>
<td>45.8</td>
<td>4.67</td>
<td>0.0147</td>
</tr>
<tr>
<td>Error</td>
<td>42</td>
<td>411.78</td>
<td>9.804</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. Total</td>
<td>44</td>
<td>503.38</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comment**

*By ignoring the Block all the variation due to differences in Subjects goes into the Error term, inflating the MSErro and making differences among Treatments (Methods) harder to detect.*
Compute the LSD for comparing the Method Means.

\[ LSD = t^* \sqrt{MSE_{Error}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \]

\[ LSD = 2.01808 \sqrt{9.804} \frac{2}{\sqrt{15}} \]

\[ = 2.01808(1.1433) = 2.307 \]

Levels not connected by the same letter are significantly different.

With the incorrect analysis, the difference between Back and Staggered is no longer statistically significant.

A major advantage of blocking is the reduction in the MSE error achieved by separating out the variation due to differences in experimental material.

If the MSE error is reduced, it is easier to detect differences in Treatment means.

This advantage disappears if the experimental material is homogeneous (uniform). Making blocks with already uniform material wastes degrees of freedom that could be used to estimate random error variation.
Lecture 25: More on Block Designs

Alternative Block Design

*Instead of reusing 15 men who have had knee replacement surgery, suppose we sort 45 men into 5 blocks of 9 men according to their Body Mass Index (BMI).

Alternative Block Design

*This requires more men.
*There is some variation within each block because not all men within a block have the same BMI.
*There are 3 men in each combination of BMI and Method, replication of Block by Treatment combinations.

Alternative Block Design

*The data from the original experiment is used in this example to show what happens when you make fewer blocks and have several individuals in each Block by Treatment combination.

Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>2</td>
<td>91.60</td>
<td>45.8</td>
<td>20.82</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Block</td>
<td>4</td>
<td>328.17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>38</td>
<td>83.61</td>
<td>2.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. Total</td>
<td>44</td>
<td>503.38</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Test of Hypothesis

*H₀: all method effects are 0
*Hₐ: some method effects are not 0
*F = 20.82, P-value < 0.0001

Test of Hypothesis

*Because the P-value is so small, we should reject the null hypothesis and conclude that some of the method effects are not zero.
Lecture 25: More on Block Designs

**Multiple Comparisons**

- Compute the LSD for comparing the Method Means.

\[
LSD = t^* \sqrt{MSE_{\text{Error}}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}
\]

\[
LSD = 2.02439 \sqrt{2.20} \sqrt{\frac{2}{15}} = 2.02439(0.5416) = 1.096
\]

- With the alternative Block design the MSEError is a little larger.
- The difference between Back and Staggered is not statistically significant.

**Multiple Comparisons**

<table>
<thead>
<tr>
<th>Level</th>
<th>Mean</th>
<th>Letter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutral</td>
<td>24.1</td>
<td>A</td>
</tr>
<tr>
<td>Back</td>
<td>21.7</td>
<td>B</td>
</tr>
<tr>
<td>Staggered</td>
<td>20.7</td>
<td>B</td>
</tr>
</tbody>
</table>

Levels not connected by the same letter are significantly different.

**The Rest of the Story**

- The “Error” term actually has two components.
- Lack of consistency of treatment effects.
- Random error due to differences in individuals treated the same.

**Lack of Consistency**

- This is the Block by Method interaction.
- \(2 \times 4 = 8\) df
- JMP calls this Lack of Fit.

**Random Error**

- Within each of the 15 Block by Method combination there are 3 individuals (2 df).
- \(15 \times 2 = 30\) df.
- JMP calls this Pure Error.
Lecture 25: More on Block Designs

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lack of Fit</td>
<td>8</td>
<td>20.60</td>
<td>2.575</td>
<td>1.23</td>
<td>0.3182</td>
</tr>
<tr>
<td>Pure Error</td>
<td>30</td>
<td>63.01</td>
<td>2.100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Error</td>
<td>38</td>
<td>83.61</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The F test for Lack of Fit is testing to see if there is a statistically significant interaction between the Blocks (BMI) and the Treatments (Methods).

*F = 1.23, P-value = 0.3183
*Because the P-value is not small (> 0.05) there is no statistically significant lack of fit (interaction).

*If the Lack of Fit were statistically significant, the MSError would be inflated.
*This would make it difficult to see differences among treatments.

*If there is no statistically significant interaction the effects of Methods are consistent across all Blocks.

*There still could be treatment effects it is just that those effects would not be the same (consistent) across all blocks.