Lecture 19: Factorial Analysis - Continued

Recall Experiment

*Formal Analysis
  * 79.3% of the variation in recall can be explained by differences among treatment means.
  * There are statistically significant differences among some of the treatment means (F = 13.785, P-value < 0.0001).

Recall Experiment

*We could follow up with comparisons of pairs of treatment means using a Tukey HSD.

Recall Experiment

*Treatments are made up of levels of two factors: Reinforcement and Isolation Time.
  * Can we explain treatment differences in terms of these factors and their interaction?

Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>5</td>
<td>896.0</td>
<td>179.2</td>
<td>13.785</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>Error</td>
<td>18</td>
<td>234.0</td>
<td>13.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. Total</td>
<td>23</td>
<td>1130.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

JMP

*In the JMP output there are Parameter Estimates and Effect Tests that quantify and test the effects of the factors and their interaction.

Parameter Estimates

*Reinforcement – JMP gives
  * Reinforce[N] = –2
  * Because estimated effects have to add to zero
  * Reinforce[V] = +2
Lecture 19: Factorial Analysis - Continued

Parameter Estimates

* Isolation Time – JMP gives
  * IsoTime[20] = -2.5
  * IsoTime[40] = 4.0
  * Because estimated effects have to add to zero
  * IsoTime[60] = -1.5

Parameter Estimates

* Interaction – JMP gives
  * Reinforce[N]*IsoTime[20] = +2.5
  * Reinforce[N]*IsoTime[40] = +4.5

Estimated Interaction Effects

<table>
<thead>
<tr>
<th>IsoTime[20]</th>
<th>IsoTime[40]</th>
<th>IsoTime[60]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforce[N]</td>
<td>+2.5</td>
<td>+4.5</td>
</tr>
<tr>
<td>Reinforce[V]</td>
<td>-2.5</td>
<td>-4.5</td>
</tr>
</tbody>
</table>

The estimated effects have to add to zero across rows and down columns.

Sum of Squares

* The sum of squares for a factor or interaction can be calculated by taking the weighted sum of squares estimated effects.

Sums of Squares

* SS_Reinforce = 12(-2)^2 + 12(+2)^2 = 96.0
* SS_Isotime = 8(-2.5)^2 + 8(+4)^2 + 8(-1.5)^2 = 196.0

* SS_Reinforce*IsoTime = 4(+2.5)^2 + 4(+4.5)^2 + 4(-7.0)^2 + 4(-2.5)^2 + 4(-4.5)^2 + 4(+7.0)^2 = 604.0
Lecture 19: Factorial Analysis - Continued

### Full Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforcement</td>
<td>1</td>
<td>96.0</td>
<td>96.0</td>
<td>7.38</td>
</tr>
<tr>
<td>Isolation Time</td>
<td>2</td>
<td>196.0</td>
<td>98.0</td>
<td>7.54</td>
</tr>
<tr>
<td>Reinforce*IsoTime</td>
<td>2</td>
<td>604.0</td>
<td>301.0</td>
<td>23.15</td>
</tr>
<tr>
<td>Error</td>
<td>18</td>
<td>234.0</td>
<td>13.0</td>
<td></td>
</tr>
<tr>
<td>C. Total</td>
<td>23</td>
<td>1130.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Reinforcement

\[ H_0: \alpha_N = \alpha_V = 0 \]
\[ H_A: \alpha_N \neq \alpha_V \neq 0 \]

*F = 7.38, P-value = 0.0141, because the P-value is so small (<0.05) there are statistically significant effects due to Reinforcement.*

Because there are only two levels of Reinforcement, the two Reinforcement means are significantly different.

Verbal Reinforcement produces a higher mean recall than No Reinforcement.

### Isolation Time

\[ H_0: \beta_{20} = \beta_{40} = \beta_{60} = 0 \]
\[ H_A: \text{some } \beta_j \text{ is not zero} \]

*F = 7.54, P-value = 0.0042, because the P-value is so small (<0.05) there are statistically significant effects due to Isolation Time.*

Follow up with pair-wise comparisons of the three Isolation Time means.

\[ LSD = t^* \left( \sqrt{MS_{\text{Error}}} \right) \left( \frac{1}{n_i} + \frac{1}{n_j} \right) \]
\[ = 2.10092 \left( \sqrt{13.0} \right) \left( \frac{1}{2} + \frac{1}{2} \right) \]
\[ = 2.10092 \times 1.80278 = 3.79 \]

Comparison | Difference in Means | > LSD? |
---|---------------------|--------|
20 to 40   | 28.0 – 21.5 = 6.5   | Yes    |
20 to 60   | 22.5 – 21.5 = 1.0   | No     |
40 to 60   | 28.0 – 22.5 = 5.5   | Yes    |
Lecture 19: Factorial Analysis - Continued

**Isolation Time**
- The 40 minute Isolation Time is statistically different from both the 20 minute and 60 minute Isolation Times.
- There is no statistically significant difference between the 20 minute and 60 minute Isolation Times.

**Interaction**

\[ H_0: \text{all } \beta_{ij} = 0 \]
\[ H_A: \text{some } \beta_{ij} \text{ are not zero} \]
- \( F = 23.15, \text{ P-value } < 0.0001 \), because the P-value is so small (<0.05) there is a statistically significant interaction between Reinforcement and Isolation Time.

**Recall LS Means**
- With Verbal Reinforcement as Isolation Time increases, average Recall increases.
- With No Reinforcement as Isolation Time increases, average Recall first increases then decreases, on average.

**Interaction**

- The effect of Isolation Time on Recall is different for different levels of Reinforcement.

**Interaction**

- The interaction is most apparent for the change from 40 to 60 minutes of Isolation Time.