Question
Is annual carbon dioxide concentration related to annual global temperature?

Response variable, $Y$.
- Annual global temperature ($^\circ$ C).
- Explanatory (predictor) variable, $x$.
  - Annual atmospheric CO$_2$ concentration.

Regression model
$Y = \mu_{y|x} + \varepsilon$

- $Y$ represents a value of the response variable.
- $\mu_{y|x}$ represents the population mean response for a given value of the explanatory variable, $x$.
- $\varepsilon$ represents the random error.
Linear Model

\[ Y = \mu_{y|x} + \varepsilon = \beta_0 + \beta_1 x + \varepsilon \]

- \( \beta_0 \): The Y-intercept parameter.
- \( \beta_1 \): The slope parameter.

Conditions

- The relationship is linear.
- The random error term, \( \varepsilon \), is
  - Independent
  - Identically distributed
  - Normally distributed with standard deviation, \( \sigma \).

![Graph](image)
Describe the plot.

- Direction – positive/negative.
- Form – linear/non-linear.
- Strength.
- Unusual points?

Method of Least Squares

- Find estimates of $\beta_0$ and $\beta_1$ such that the sum of squared vertical deviations from the estimated straight line is the smallest possible.

Least Squares Estimates

\[
\hat{\beta}_1 = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}
\]

\[
\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}
\]

\[
\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x
\]
Bivariate Fit of Temp By CO2

Linear Fit

\[
\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x
\]

Predicted Temp = 9.8815 + 0.012584*CO₂

Interpretation

Estimated Y-intercept.

This does not have an interpretation within the context of the problem. Having no CO₂ in the atmosphere is not reasonable given the data.
Interpretation

- Estimated slope.
- For each additional 1 ppmv of CO₂, the annual global temperature goes up 0.012584 °C, on average.

How Strong?

- The strength of a linear relationship can be measured by $R^2$, the coefficient of determination.
- RSquare in JMP output.
How Strong?

\[ R^2 = \frac{SS_{Model}}{SS_{Total}} \]

\[ R^2 = \frac{0.80145}{0.99450} = 0.806 \]

Interpretation

"80.6% of the variation in the global temperature can be explained by the linear relationship with carbon dioxide concentration. 19.4% is unexplained."

Interpretation

"There is a fairly strong positive linear relationship between carbon dioxide concentration and global temperature. Cause and effect?"
Cause and Effect?
- There is a strong positive linear relationship between the number of 2nd graders in communities and the number of crimes committed in those communities.

Connection to Correlation
- If you square the correlation coefficient, \( r \), relating carbon dioxide to global temperature you get \( R^2 \), the coefficient of determination.
  \[
  r = \pm \sqrt{R^2} = \pm \sqrt{0.806} = \pm 0.898
  \]

Connection to Correlation
- \( \hat{\beta}_1 = r \left( \frac{s_y}{s_x} \right) \)
  - \( s_y \) is the standard deviation of the y values
  - \( s_x \) is the standard deviation of the x values