Stat 401 B – Lecture 29

Simple Linear Regression

- Example - mammals
- Response variable: gestation (length of pregnancy) days
- Explanatory: brain weight

1

Simple Linear Regression

- Predicted Gestation = 85.25 + 0.30*Brain Weight
- R^2 = 0.372, so only 37.2% of the variation in gestation is explained by the linear relationship with brain weight.

2

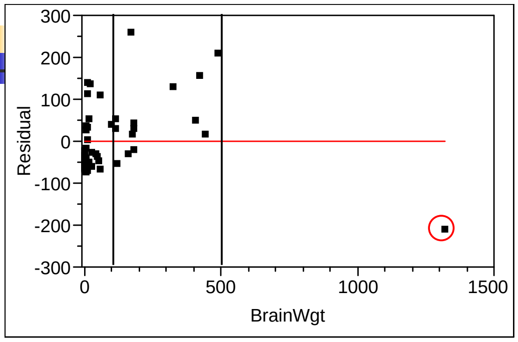
Regression Residuals

$$\text{residual} = y - \hat{y}$$

- Those observations that do not follow the general trend will have residuals that are far from zero, either positive or negative.

3

Stat 401 B – Lecture 29



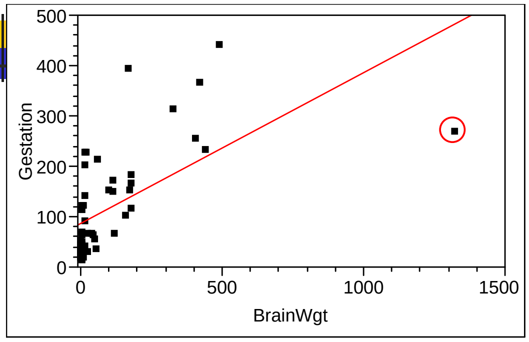
Pattern in the Residuals

- Brain Weight between 0 g and 100 g
  - 25 negative, 10 positive residuals
- Brain Weight between 100 g and 500 g
  - 3 negative, 11 positive residuals

Pattern in the Residuals

- There may be an positive relationship between brain weight and the residuals for brain weights less than 500 g.
- This pattern could be due to the large brain weight mammal with a relatively short gestation.

Stat 401 B – Lecture 29



7

Leverage

- A point with an extreme value for the explanatory variable can exert leverage on where the regression line will go.
- The leverage is quantified by something called the "hat" value.

8

Leverage in SLR

- In simple linear regression there is an equation for the "hat" value, h.

$$h = \frac{1}{n} + \frac{(x_o - \bar{x})^2}{\sum (x - \bar{x})^2}$$

9

Stat 401 B – Lecture 29

Comment

- Leverage is a summarizing something about the explanatory variable, *x*.
- Brain Weight:
  - Mean: $\bar{x} = 107.2524$
  - Std Dev: $s = 216.35854$

10

Sum of Squares for *x*

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$(n-1)s^2 = \sum (x - \bar{x})^2$$

$$(49)(216.35884)^2 = 2293739.874$$

11

Leverage for "Man"

$$x_0 = 1320$$

$$h = \frac{1}{50} + \frac{(1320 - 107.2524)^2}{2293739.874}$$

$$h = 0.02 + 0.641 = 0.661$$

12

Rule of Thumb

- High Leverage Value if

$$h > 2\left(\frac{p+1}{n}\right)$$

- n = 50, p = 1,

$$2\left(\frac{p+1}{n}\right) = 2\left(\frac{2}{50}\right) = 0.08$$

13

Comment

- The leverage for "Man", 0.661, is greater than 0.08.
- This indicates that the brain weight for "Man" is unusual and that "Man" is a high leverage point.

14

Statistical Significance

- Test statistic

$$F = \frac{\left(h - \frac{1}{n}\right)\Big/ p}{(1-h)/(n-p-1)}$$

- Compute the P-value for an F distribution with p and $(n - p - 1)$ degrees of freedom.

15

Stat 401 B – Lecture 29

Statistical Significance

- "Man"
  - h = 0.661, p = 1, n = 50

$$F=\frac{\left(h-\frac{1}{n}\right)\Big/p}{(1-h)/(n-p-1)}=\frac{(0.661-0.02)/1}{(1-0.661)/48}=90.76$$

Computing a P-value

- JMP – Col – Formula
- (1 – F Distribution(F,p,n-p-1))
- For our example
  - F=90.76, p=1, n-p-1=48
  - P-value = 0.000000000001208

Conclusion – "Man"

- The P-value is much less than 0.001 (the Bonferroni corrected cutoff), therefore "Man" is a statistically significant high leverage point.

Stat 401 B – Lecture 29

Other High Leverage Points

- The only other mammal with leverage above 0.08 is the Okapi.
- h = 0.084
- F = 3.354
- P-value = 0.07325

Conclusion - Okapi

- The P-value is greater than 0.001, although the Okapi has high leverage, that leverage is not statistically significant.

"Man"

- Extreme negative residual but that residual is not statistically significant.
- The extreme brain weight of "man" creates high leverage that is statistically significant.