Stat 401 B – Lecture 20

Quadratic Model

- In order to account for curvature in the relationship between an explanatory and a response variable, one often adds the square of the explanatory variable to the simple linear model.

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Quadratic Model

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \varepsilon$$

- Conditions on ε
  - Independent
  - Identically distributed
  - Normally distributed with common standard deviation, σ

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Example

- Response, Y: Population of the U.S. (millions)
- Explanatory, X: Year the census was taken.

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Quadratic Model

- Predicted Population = 21006.1 –23.3785*Year + 0.00651*$Year^2$
- We cannot interpret the estimated slope coefficients because we cannot change Year by 1 while holding $Year^2$ constant.

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Model Utility

- F=8050.89, P-value<0.0001
- The small P-value indicates that the quadratic model relating population to Year and $Year^2$ is statistically significant (useful).

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Statistical Significance

- Year (added to $Year^2$)
  - t=–33.48, P-value<0.0001
- The P-value is small, therefore the addition of Year is statistically significant.

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Statistical Significance

- $Year^2$ (added to Year)
  - t=35.22, P-value<0.0001
- The P-value is small, therefore the addition of $Year^2$ is statistically significant.

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Quadratic Model

- R^2=0.999 or 99.9% of the variation in population can be explained by the quadratic model.
- RMSE=2.77

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Summary - Quadratic

- The model is useful.
- Each term is a statistically significant addition.
- 99.9% of the variation in population is explained by the quadratic model.

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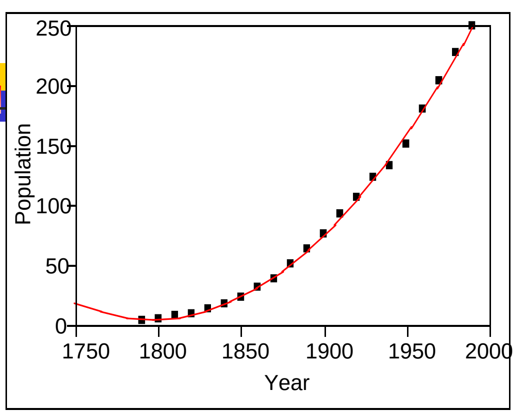
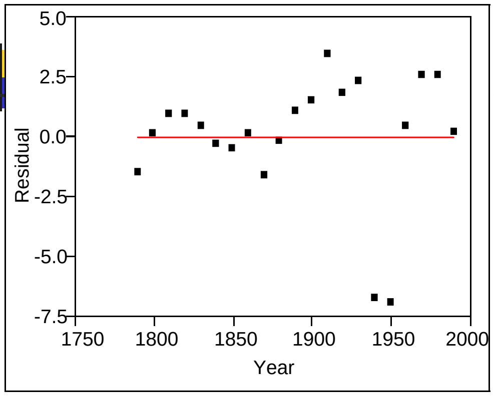
Prediction

- Year 2000
  - Predicted Population = 21006.1 – 23.3785(2000) + 0.0065063*$(2000)^2$ = 274.3 million
- Not bad as the actually figure in 2000 was 281.422 million.

Prediction

- Year 1800
  - Predicted Population = 21006.1 – 23.3785(1800) + 0.0065063*$(1800)^2$ = 5.212 million
- Very close to the actual value of 5.308 million





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Plot of Residuals

- The residuals wiggle around the zero line. Hard to say whether this is a pattern or not.
- The residuals for 1940 and 1950 stick out. The quadratic model over predicts for these years.

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Can we do better?

- Could try higher order polynomial terms like $Year^3$ or $Year^4$.
- $Year^3$ is not statistically significant in a cubic model.
- $Year^4$ is not statistically significant in a quartic model.

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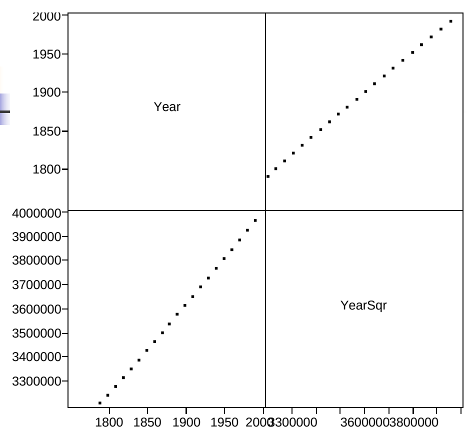
Quadratic Model

- There is still the issue of trying to interpret the coefficients in the quadratic model.
- Again, creating a new explanatory variable, Year^2, has introduced multicollinearity into the quadratic model.



Correlation

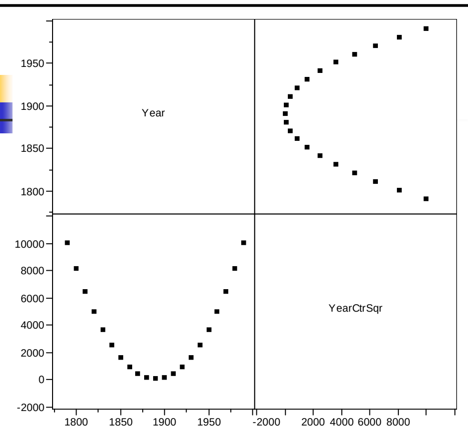
- Year and Year^2
- Correlation: r = 0.9999
- For the values that Year takes on, there is an extremely strong positive linear correlation with Year^2.

Centering

- Center Year by subtracting off the mean before constructing the squared term in the quadratic model.
- Mean year is 1890.

Quadratic Model

- Predicted Population =
  $-2235.197 + 1.215*\text{Year} + 0.00651*(\text{Year} - 1890)^2$
- Note that the estimated slope for year is exactly the same as in the simple linear model.



Correlation

- Year and (Year – 1890)^2
- Correlation: r = –0.0000
- For the values that Year takes on, there is no linear correlation with (Year – 1890)^2.

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Centering

- Centering has completely removed the multicollinearity resulting from the inclusion of the quadratic term in the quadratic model.

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Quadratic Model

- Predicted Population = 61.926 + 1.215*(Year – 1890) + 0.00651*(Year – 1890)^2
- The predicted population in 1890 is 61.926 million.

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Quadratic Model

- Predicted Population = 61.926 + 1.215*(Year – 1890) + 0.00651*(Year – 1890)^2
- For each additional year, the population goes up, on average, 1.215 million.

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Quadratic Model

- Predicted Population = 61.926 + 1.215*(Year – 1890) + 0.00651*(Year – 1890)^2
- In addition to the average change per year, there is a bigger adjustment to this rate of change the further away you are from 1890.

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One Year Change

- 1880: Pred = 50.427 million
- 1890: Pred = 61.926 million
  - Difference of 11.499
- 1980: Pred = 224.007
- 1990: Pred = 248.526
  - Difference of 24.519

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