



Stat 401 B – Lecture 17



Prices of Antique Clocks

- Antique clocks are sold at auction. We wish to investigate the relationship between the age of the clock and the auction price and whether that relationship changes if there are different numbers of bidders at the auctions.


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Price of Antique Clocks

- Response: price (pounds sterling)
- Explanatory: age (years)
- Explanatory: number of bidders

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Interaction

- There is no interaction between two explanatory variables if the relationship between the response and an explanatory variable is not different for different values of the second explanatory variable.

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Price and Age

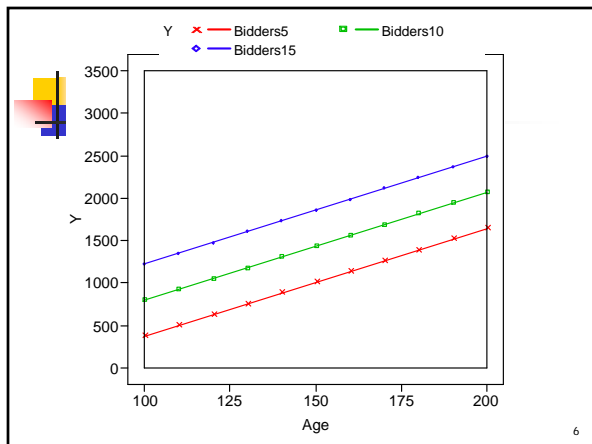
- One would expect that as the age of the clock increases the price paid for it at auction would increase.

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No interaction

- If there is no interaction between Age and Bidders, the linear relationship between Price and Age will be the same (have the same slope coefficient) regardless of how many Bidders are at an auction.

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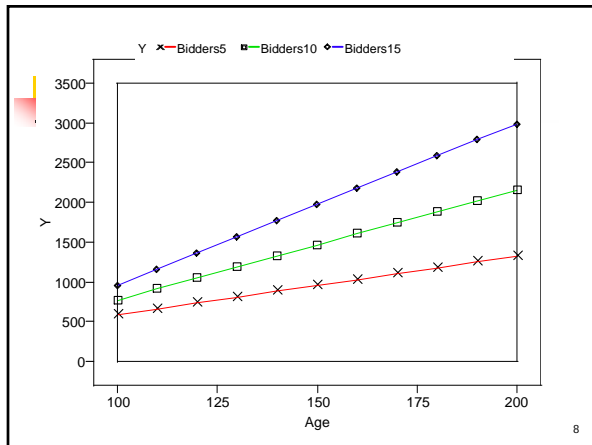


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Interaction

- If there is interaction between Age and Bidders, the linear relationship between Price and Age will change (have different values for the slope coefficient) as the number of Bidders at auctions change.

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No Interaction Model

$$\text{Price} = \beta_0 + \beta_1 * \text{Age} + \beta_2 * \text{Bidders} + \varepsilon$$

- Note: Putting in any number of Bidders will not change what multiplies Age. Changing the number of Bidders will change the level of the regression line.

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Interaction Model

$$\text{Price} = \beta_0 + \beta_1 * \text{Age} + \beta_2 * \text{Bidders} \\ + \beta_3 * (\text{Age} * \text{Bidders}) + \varepsilon$$

- Note: Changing the number of Bidders will change what multiplies Age as well as the level of the regression line.

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Interaction Model – 5 Bidders

$$\text{Price} = \beta_0 + \beta_1 * \text{Age} + \beta_2 * 5 \\ + \beta_3 * (\text{Age} * 5) + \varepsilon \\ \text{Price} = (\beta_0 + 5\beta_2) + (\beta_1 + 5\beta_3) * \text{Age} + \varepsilon$$


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Interaction Model – 10 Bidders

$$\text{Price} = \beta_0 + \beta_1 * \text{Age} + \beta_2 * 10 \\ + \beta_3 * (\text{Age} * 10) + \varepsilon \\ \text{Price} = (\beta_0 + 10\beta_2) + (\beta_1 + 10\beta_3) * \text{Age} + \varepsilon$$

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Interaction

- The slope coefficient that multiplies Age changes as the number of Bidders changes.


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Which Model is Better?

- The two models differ because one includes a cross product (interaction) term – Age*Bidders.
- If the Age*Bidders term is statistically significant, then there is statistically significant interaction between age and bidders.

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Which Model is Better?

- The two models differ because one includes a cross product (interaction) term – Age*Bidders.
- If the Age*Bidders term is not statistically significant, then there is no statistically significant interaction between age and bidders.

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Clock Auction Data

- The auction price for 32 antique grandfather clocks is obtained along with the ages of the clocks and the number of bidders at the auctions when the clock was sold.


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Simple Linear Regression

- Predicted Price = $-191.6576 + 10.479 \cdot \text{Age}$
- For each additional year of age, the price of the clock increases by 10.479 pounds sterling, on average.

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Simple Linear Regression

- There is not reasonable interpretation of the intercept because an antique clock cannot have an age of zero.

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Model Utility

- $F=34.273$, $P\text{-value}<0.0001$
- The small P-value indicates that the simple linear model using age is useful in explaining variability in the prices of antique clocks.

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Statistical Significance

- Parameter Estimates
 - $t=5.85$, $P\text{-value}<0.0001$
- Effect Tests
 - $F=34.273$, $P\text{-value}<0.0001$
- The P-value is small, therefore there is a statistically significant linear relationship between price and age.


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Simple Linear Model

- $R^2=0.533$ or 53.3% of the variation in price can be explained by the linear relationship with age.
- $RMSE=273.03$

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Simple Linear Model

- The simple linear model is useful and explains over 50% of the variation in price.
- Not bad.
- Can we do better?


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No Interaction Model

- Predicted Price = $-1336.722 + 12.736 \cdot \text{Age} + 85.815 \cdot \text{Bidders}$
- For auctions with the same number of bidders, for a one year increase in age, the price of the clock increases 12.736 pounds sterling, on average.

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No Interaction Model

- Predicted Price = $-1336.722 + 12.736 \cdot \text{Age} + 85.815 \cdot \text{Bidders}$
- For clocks of the same age, increase the number of bidders by one, the price of the clock increases 85.815 pounds sterling, on average.

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Model Utility

- $F=120.65$, $P\text{-value}<0.0001$
- The small P-value indicates that the model using Age and Bidders is useful in explaining variability in the prices of antique clocks.

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Statistical Significance

- Age (added to Bidders)
 - $t=14.11$, $P\text{-value}<0.0001$
 - $F=199.205$, $P\text{-value}<0.0001$
- The P-value is small, therefore Age adds significantly to the model that already contains Bidders.


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Statistical Significance

- Bidders (added to Age)
 - $t=9.86$, $P\text{-value}<0.0001$
 - $F=97.166$, $P\text{-value}<0.0001$
- The P-value is small, therefore Bidders adds significantly to the model that already contains Age.


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 **No Interaction Model**


- $R^2=0.893$ or 89.3% of the variation in price can be explained by the no interaction model.
- $RMSE=133.14$

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 **No Interaction Model**

- Number of Bidders = 5
 - Predicted Price = $-907.647 + 12.736 * \text{Age}$
- Number of Bidders = 10
 - Predicted Price = $-478.572 + 12.736 * \text{Age}$

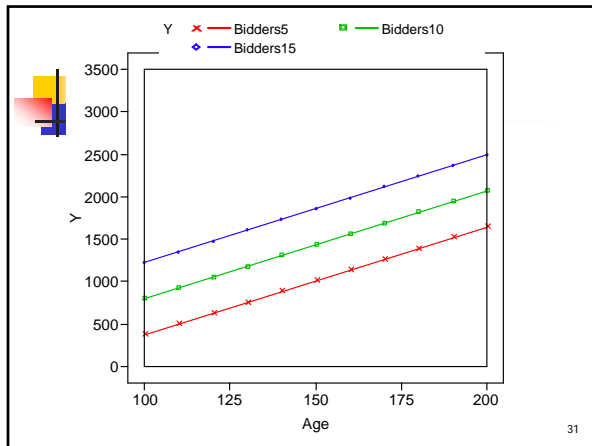
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 **No Interaction Model**

- Number of Bidders = 15
 - Predicted Price = $-49.497 + 12.736 * \text{Age}$
- The slope estimate for Age is the same for all numbers of bidders.

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Summary

- Simple linear regression of price on age. Not bad.
- No interaction model including age and bidders. Better.

Interaction Model

- Predicted Price = $322.754 + 0.873 \cdot \text{Age} - 93.410 \cdot \text{Bidders} + 1.298 \cdot \text{Age} \cdot \text{Bidders}$
- Something looks funny because the sign of the estimated slope for Bidders is negative.


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Interaction Model

- Cannot interpret any of the estimated slope coefficients.
- Cannot change one variable and hold the other two constant.
- If you change Age, you cannot hold Age*Bidders constant.


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Model Utility

- $F=195.19$, $P\text{-value}<0.0001$
- The small P-value indicates that the model using Age, Bidders and Age*Bidders is useful in explaining variability in the prices of antique clocks.

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Statistical Significance

- Age*Bidders (added to Age, Bidders)
 - $t=6.15$, $P\text{-value}<0.0001$
 - $F=37.83$, $P\text{-value}<0.0001$
- The P-value is small, therefore the interaction term Age*Bidders adds significantly to the no interaction model.

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Interaction Model

- $R^2=0.954$ or 95.4% of the variation in price can be explained by the interaction model.
- $RMSE=88.37$


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Interaction Model

- Number of Bidders = 5
 - Predicted Price = $-144.296 + 7.363 \cdot \text{Age}$
- Number of Bidders = 10
 - Predicted Price = $-611.346 + 13.853 \cdot \text{Age}$

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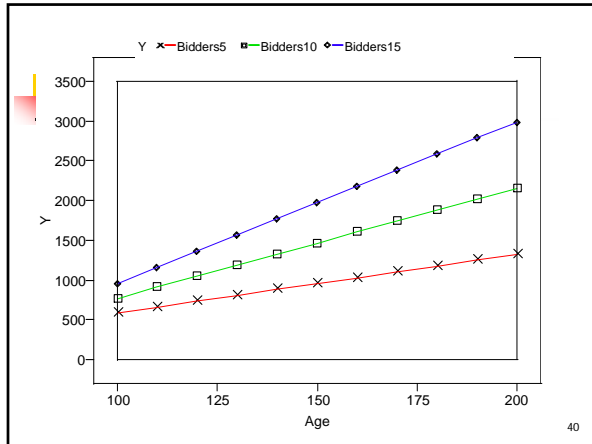


Interaction Model

- Number of Bidders = 15
 - Predicted Price = $-1078.396 + 20.343 \cdot \text{Age}$
- The slope estimate for Age changes as the number of bidders changes.

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Something Unusual

- The interaction model is doing an even better job than the no interaction model.
- The test for Age in the interaction model is not statistically significant.

Something Unusual

- If you Remove Age from the interaction model, JMP forces you to remove Age*Bidders as well.
- JMP forces you to include all individual terms if the interaction term is included.
