Tennis Ball Experiment

An experiment is performed to see how differing amounts of three key components; silica, silane and sulfur affect the bounciness of a tennis ball. A wide variety of values for each component are used. Ultimately, the combination of components that produces a bounce response of 450 is desired. The data appear below.

<table>
<thead>
<tr>
<th>Silica</th>
<th>Sulfur</th>
<th>Silane</th>
<th>Bounce</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>1.8</td>
<td>50</td>
<td>570</td>
</tr>
<tr>
<td>0.7</td>
<td>2.8</td>
<td>50</td>
<td>285</td>
</tr>
<tr>
<td>1.7</td>
<td>1.8</td>
<td>50</td>
<td>260</td>
</tr>
<tr>
<td>1.7</td>
<td>2.8</td>
<td>50</td>
<td>433</td>
</tr>
<tr>
<td>1.2</td>
<td>1.8</td>
<td>40</td>
<td>422</td>
</tr>
<tr>
<td>1.2</td>
<td>1.8</td>
<td>60</td>
<td>351</td>
</tr>
<tr>
<td>1.2</td>
<td>2.8</td>
<td>40</td>
<td>278</td>
</tr>
<tr>
<td>1.2</td>
<td>2.8</td>
<td>60</td>
<td>392</td>
</tr>
<tr>
<td>0.7</td>
<td>2.3</td>
<td>40</td>
<td>451</td>
</tr>
<tr>
<td>1.7</td>
<td>2.3</td>
<td>40</td>
<td>372</td>
</tr>
<tr>
<td>0.7</td>
<td>2.3</td>
<td>60</td>
<td>474</td>
</tr>
<tr>
<td>1.7</td>
<td>2.3</td>
<td>60</td>
<td>394</td>
</tr>
<tr>
<td>1.2</td>
<td>2.3</td>
<td>50</td>
<td>398</td>
</tr>
<tr>
<td>1.2</td>
<td>2.3</td>
<td>50</td>
<td>394</td>
</tr>
<tr>
<td>1.2</td>
<td>2.3</td>
<td>50</td>
<td>396</td>
</tr>
</tbody>
</table>

A Response Surface Model includes each of the explanatory variables; silica, sulfur and silane; all pair-wise interaction variables; silica*sulfur, silica*silane, sulfur*silane; and all squared explanatory variables; silica^2, sulfur^2, silane^2. Centering the variables in the interaction and squared terms tends to reduce multicollinearity.

Response: Bounce

Summary of Fit

RSquare 0.999771
RSquare Adj 0.999543
Root Mean Square Error 1.700032
Mean of Response 391.3333
Observations (or Sum Wgts) 15

Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>7</td>
<td>88453.103</td>
<td>12636.2</td>
<td>4372.207</td>
</tr>
<tr>
<td>Error</td>
<td>7</td>
<td>20.231</td>
<td>2.9</td>
<td>Prob &gt; F</td>
</tr>
<tr>
<td>C. Total</td>
<td>14</td>
<td>88473.333</td>
<td>&lt;.0001*</td>
<td></td>
</tr>
</tbody>
</table>
### Parameter Estimates

| Term                        | Estimate  | Std Error | t Ratio | Prob>|t|  |
|-----------------------------|-----------|-----------|---------|------|---|
| Intercept                   | 561.07885 | 4.40724   | 127.31  | <.0001* |
| Silica                      | –80.25    | 1.202104  | –66.76  | <.0001* |
| Sulfur                      | –53.75    | 1.202104  | –44.71  | <.0001* |
| Silane                      | 1.1       | 0.060105  | 18.30   | <.0001* |
| (Silica-1.2)*(Silica-1.2)   | 105.92308 | 3.528414  | 30.02   | <.0001* |
| (Silica-1.2)*(Sulfur-2.3)   | 458       | 3.400065  | 134.70  | <.0001* |
| (Sulfur-2.3)*(Sulfur-2.3)   | –142.0769 | 3.528414  | –40.27  | <.0001* |
| (Sulfur-2.3)*(Silane-50)    | 9.25      | 0.170003  | 54.41   | <.0001* |

### Contour Profiler

![Contour Plot](image)

<table>
<thead>
<tr>
<th>Horiz</th>
<th>Vert</th>
<th>Factor</th>
<th>Current X</th>
</tr>
</thead>
<tbody>
<tr>
<td>☰</td>
<td>☰</td>
<td>Silica</td>
<td>1.055</td>
</tr>
<tr>
<td>☰</td>
<td>☰</td>
<td>Sulfur</td>
<td>2</td>
</tr>
<tr>
<td>☰</td>
<td>☰</td>
<td>Silane</td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Response</th>
<th>Contour</th>
<th>Current Y</th>
<th>Lo Limit</th>
<th>Hi Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bounce</td>
<td></td>
<td>450</td>
<td>450.02821</td>
<td>.</td>
</tr>
</tbody>
</table>