Review

- The equation of a straight line
- \( y = mx + b \)
  - \( m \) is the slope – the change in \( y \) over the change in \( x \) – or rise over run.
  - \( b \) is the \( y \)-intercept – the value where the line cuts the \( y \) axis.

\[ y = 3x + 2 \]

- \( x = 0 \quad \rightarrow \quad y = 2 \) (y-intercept)
- \( x = 3 \quad \rightarrow \quad y = 11 \)
- Change in \( y \) (+9) divided by the change in \( x \) (+3) gives the slope, 3.
Linear Regression

- Example: Body mass (kg) and Bite force (N) for Canidae.
  - $y$, Response: Bite force (N)
  - $x$, Explanatory: Body mass (kg)
  - Cases: 28 species of Canidae.

Correlation Coefficient

- Body mass and Bite force
  
  \[ r = \frac{\sum z_x z_y}{n - 1} = \frac{26.4796}{27} \]

- $r = 0.9807$

- There is a strong correlation, linear association, between the body mass and bite force for the various species of Canidae.
Linear Model

- The linear model is the equation of a straight line through the data.
- A point on the straight line through the data gives a predicted value of \( y \), denoted \( \hat{y} \).

Residual

- The difference between the observed value of \( y \) and the predicted value of \( y \), \( \hat{y} \), is called the residual.
- Residual = \( y - \hat{y} \)
Line of “Best Fit”

- There are lots of straight lines that go through the data.
- The line of “best fit” is the line for which the sum of squared residuals is the smallest – the least squares line.

\[ \hat{y} = b_0 + b_1 x \]

Least squares slope:  
\[ b_1 = r \frac{s_y}{s_x} \]

intercept:  
\[ b_0 = \bar{y} - b_1 \bar{x} \]

Least Squares Estimates

<table>
<thead>
<tr>
<th>Body mass, $x$</th>
<th>Bite Force, $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x} = 9.207$ kg</td>
<td>$\bar{y} = 154.029$ N</td>
</tr>
<tr>
<td>$s_x = 8.016$ kg</td>
<td>$s_y = 109.760$ N</td>
</tr>
<tr>
<td>$r = 0.9807$</td>
<td></td>
</tr>
</tbody>
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