Two Independent Samples

- Do males and females at I.S.U. spend the same amount of time, on average, at the Lied Recreation Athletic Center?

Inference

1. Female
2. Male

Samples

2. Males: 52, 75, 74, 68, 93, 77, 41, 87, 72, 53, 84, 65, 66, 69, 62
Comment

• This sample of I.S.U. females spends, on average, 13.33 minutes less time at the Lied Recreation Athletic Center than this sample of I.S.U. males.

Confidence Interval: $\mu_1 - \mu_2$

$$(\bar{y}_1 - \bar{y}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$t^*$ from Table T,
df = smaller of $(n_1 - 1, n_2 - 1)$
Finding $t^*$

- Use Table T.
- Confidence Level in last row.
- df = smaller of $(n_1 - 1, n_2 - 1) = 14$. 

<table>
<thead>
<tr>
<th>df</th>
<th>Confidence Levels</th>
<th>80%</th>
<th>90%</th>
<th>95%</th>
<th>98%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>2.145</td>
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<td>2</td>
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<td>14</td>
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</tbody>
</table>

\[ \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{(13.527)^2}{15} + \frac{(13.792)^2}{15}} = \sqrt{24.88} = 4.988 \]
Confidence Interval: $\mu_1 - \mu_2$

$$(\bar{y}_1 - \bar{y}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(55.87 - 69.2) \pm 2.145(4.988)$$

$- 13.33 \pm 10.70$

$- 24.03 \text{ to } - 2.63$

Interpretation

- We are 95% confident that I.S.U. females spend, on average, from 2.63 to 24.03 minutes less time at the Lied Recreation Athletic Center than I.S.U. males do.

Test of Hypothesis: $\mu_1 - \mu_2$

- Step 1: Set – up.

$H_0: \mu_1 = \mu_2 \text{ or } H_0: \mu_1 - \mu_2 = 0$

$H_A: \mu_1 \neq \mu_2 \text{ or } H_A: \mu_1 - \mu_2 \neq 0$
Test of Hypothesis: $\mu_1 - \mu_2$

- Step 2: Test Criteria.
  - Times are normally distributed.
  - Population standard deviations not known.
  - $t$-test statistic.
  - $\alpha = 0.05$

Test of Hypothesis: $\mu_1 - \mu_2$

- Step 3: Sample evidence.

$$t = \frac{(\bar{y}_1 - \bar{y}_2) - 0}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$$

<table>
<thead>
<tr>
<th>Sex=F</th>
<th>Sex=M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>55.87</td>
</tr>
<tr>
<td>Std Dev</td>
<td>13.527</td>
</tr>
<tr>
<td>n</td>
<td>15</td>
</tr>
</tbody>
</table>
\[ \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{(13.527)^2}{15} + \frac{(13.792)^2}{15}} = \sqrt{24.88} = 4.988 \]

Test of Hypothesis: \( \mu_1 - \mu_2 \)

- Step 3: Sample evidence.

\[
t = \frac{(\bar{y}_1 - \bar{y}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(55.87 - 69.20)}{4.988} = -2.672
\]

<table>
<thead>
<tr>
<th>df</th>
<th>0.20</th>
<th>0.10</th>
<th>0.05</th>
<th>0.02</th>
<th>P-value</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.345</td>
<td>1.761</td>
<td>2.145</td>
<td>2.624</td>
<td>2.672</td>
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</table>
Test of Hypothesis: $\mu_1 - \mu_2$

Step 4: Probability value
- The P-value is between 0.01 and 0.02.

Test of Hypothesis: $\mu_1 - \mu_2$

Step 5: Results.
- Reject the null hypothesis because the P-value is smaller than $\alpha = 0.05$
- The difference in mean times is not zero. Therefore, on average, females and males at I.S.U. spend different amounts of time at the Lied Recreation Athletic Center.

Comment

- This conclusion agrees with the results of the confidence interval.
- Zero is not contained in the 95% confidence interval (~24.03 mins to ~2.63 mins), therefore the difference in population mean times is not zero.
JMP

• Data in two columns.
  – Response variable:
    • Numeric – Continuous
  – Explanatory variable:
    • Character – Nominal

JMP Starter

• Basic – Two-Sample t-Test
  – Y, Response: Time
  – X, Grouping: Sex

Means and Std Deviations

<table>
<thead>
<tr>
<th>Level</th>
<th>Number</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Std Err Mean</th>
<th>Lower 95%</th>
<th>Upper 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>15</td>
<td>55.86</td>
<td>3.49</td>
<td>48.376</td>
<td>46.376</td>
<td>60.93</td>
</tr>
<tr>
<td>M</td>
<td>5</td>
<td>69.20</td>
<td>3.56</td>
<td>61.563</td>
<td>59.938</td>
<td>68.83</td>
</tr>
</tbody>
</table>

T Test

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Difference</th>
<th>Std Err Diff</th>
<th>Upper CL Diff</th>
<th>Lower CL Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degrees of Freedom</td>
<td>27.9896</td>
<td>0.0124</td>
<td>0.9938</td>
<td>0.0062</td>
</tr>
<tr>
<td>Probability</td>
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