Interpretation

• We are 95% confident that the population mean alcohol content of beer is between 4.537% and 4.987%.

Interpretation

• The population mean alcohol content of beer could be any value between 4.537% and 4.987%.
• If we repeat the procedure that produces a confidence interval, 95% of intervals produced will capture the population mean.

Testing Hypotheses

• Use sample data, in the form of a sample statistic, to support or refute hypotheses.
• How likely is it to get the sample data if a hypothesis is true?
## Testing – Step by Step

- **Step 1 – Set-Up**
  - \( \mu \) is the population mean alcohol content of beer.
  - Null hypothesis: \( H_0: \mu = 5 \)
  - Alternative hypothesis: \( H_A: \mu \neq 5 \)

- **Step 2 – Test Criteria**
  - Distribution of alcohol content is approximately normal.
  - \( \sigma \) is unknown
  - \( t \) test statistic.
  - \( \alpha = 0.05 \)

- **Step 3 – Sample Evidence**
- **Step 4 – Probability Value**
- **Step 5 – Results**
Step 3 – Sample Evidence

- Sample mean,
  \( \bar{y} = 4.762 \)
- Sample standard deviation,
  \( s = 0.314 \)

Step 3 – Sample Evidence

- Test statistic,
  \[
  t = \frac{\bar{y} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{4.762 - 5}{\left(\frac{0.314}{\sqrt{10}}\right)} = \frac{-0.238}{0.0993} = -2.397
  \]

Table T

<table>
<thead>
<tr>
<th>Two tail probability</th>
<th>0.20</th>
<th>0.10</th>
<th>0.05 P-value</th>
<th>0.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>df</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2.262</td>
<td>2.397</td>
<td>2.821</td>
<td></td>
</tr>
</tbody>
</table>

The P-value is between 0.02 and 0.05.
Step 4 – Probability Value

• Alternative hypothesis
  \( \text{H}_A: \mu \neq 5 \)
• P-value is the two tail probability, df = 9.
• Table T: P-value is between 0.02 and 0.05.

Step 5 - Results

• Reject the null hypothesis because the P-value is smaller than 0.05
• The population mean alcohol content of beer is not 5%.

Another Example

• What is the mean heart rate for all young adults?
• Could the population mean heart rate for young adults be 70 beats per minute or is it something higher?
Sample Data

- Random sample of $n = 25$ young adults.
- Heart rate – beats per minute
  70, 74, 75, 78, 74, 64, 70, 78, 81, 73
  82, 75, 71, 79, 73, 79, 85, 79, 71, 65
  70, 69, 76, 77, 66

Summary of Data

- $n = 25$
- $\bar{y} = 74.16$ beats
- $s = 5.375$ beats

Step 1 – Set up

- $\mu$ is the population mean heart rate of young adults.

  $H_0 : \mu = 70$
  $H_\alpha : \mu > 70$
Step 2 – Test Criteria

• Distribution of heart rate is approximately normal.
• \[ \sigma \] is unknown
• t test statistic.
• \[ \alpha = 0.05 \]

Step 3 – Sample Evidence

\[
t = \frac{\bar{y} - \mu_0}{s / \sqrt{n}} = \frac{74.16 - 70}{5.375 / \sqrt{25}} = 4.16 / 1.075 = 3.87
\]

Use Table T to find the P-value.

Table T

<table>
<thead>
<tr>
<th>df</th>
<th>0.10</th>
<th>0.05</th>
<th>0.025</th>
<th>0.01</th>
<th>0.005</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.064</td>
<td>2.492</td>
<td>2.797</td>
<td>3.87</td>
<td>4.997</td>
<td></td>
</tr>
</tbody>
</table>

The P-value is less than 0.005.
Step 4 – Probability Value

• Alternative hypothesis
  \(-H_A: \mu > 70\)
• P-value is the one tail probability, df = 24.
• Table T: P-value is less than 0.005.

Step 5 - Results

• Reject the null hypothesis because the P-value is less than the significance level, \(\alpha\).
• The mean heart rate for the population of young adults is more than 70 beats per minute.

Alternatives

\(H_0: \mu = \mu_0\)
\(H_A: \mu < \mu_0\), One tail prob (Pr < t)
\(H_A: \mu > \mu_0\), One tail prob (Pr > t)
\(H_A: \mu \neq \mu_0\), Two tail prob (Pr > |t|)