Normal Approximation of the Binomial

- $X$ is a Binomial random variable that counts the number of successes in $n$ independent trials with success probability $p$.
  - Mean, $\mu = np$
  - Standard deviation, $\sigma = \sqrt{np(1-p)}$

Normal Approximation of the Binomial

- For large $n$, it is difficult to calculate Binomial probabilities from the formula.
- For large $n$, the Binomial distribution is symmetric, mounded at $np$ and looks like a normal model.

Example

- 38% of people in the U.S. have O+ blood type.
- If 1,000 people, chosen at random, donate blood, what is the chance that 360 or fewer will be O+?
Example

• We want to calculate $P(X \leq 360)$
• Mean
  - $\mu = np = 1000(0.38) = 380$
• Standard deviation
  - $\sigma = \sqrt{np(1-p)} = \sqrt{1000(0.38)(1-0.38)}$
  - $\sigma = \sqrt{235.6} = 15.35$

Standardize

$$z = \frac{X - \mu}{\sigma} = \frac{360 - 380}{15.35} = \frac{-20}{15.35} = -1.30$$
Standard Normal Model

- Standard normal table handed out in class.
- Table 3: page 662 in your text.
- [http://davidmlane.com/hyperstat/z_table.html](http://davidmlane.com/hyperstat/z_table.html)

Comments

- The book suggests a continuity correction factor.
  - Add 0.5 to X for the probability of being less than or equal to X.
  - Subtract 0.5 from X for the probability of being greater than or equal to X.

Continuity Correction Factor

\[ z = \frac{X - \mu}{\sigma} \]
\[ z = \frac{360.5 - 380}{15.35} = \frac{-19.5}{15.35} = -1.27 \]