Stat 104 – Lecture 17

Normal Approximation of the Binomial

- X is a Binomial random variable that counts the number of successes in n independent trials with success probability p.
  - Mean, $\mu = np$
  - Standard deviation, $\sigma = \sqrt{np(\mathbf{1} - p)}$

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Normal Approximation of the Binomial

- For large n, it is difficult to calculate Binomial probabilities from the formula.
- For large n, the Binomial distribution is symmetric, mounded at np and looks like a normal model.

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Example

- 38% of people in the U.S. have O+ blood type.
- If 1,000 people, chosen at random, donate blood, what is the chance that 360 or fewer will be O+?

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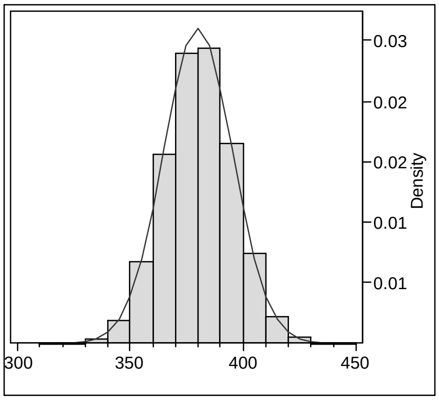

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Example

- We want to calculate $P(X \leq 360)$
- Mean
  - $\mu = np = 1000(0.38) = 380$
- Standard deviation
  - $\sigma = \sqrt{np(1-p)} = \sqrt{1000(0.38)(1-0.38)}$
    $\sigma = \sqrt{235.6} = 15.35$

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Standardize

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{360 - 380}{15.35} = \frac{-20}{15.35} - 1.30$$

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Standard Normal Model

- Standard normal table handed out in class.
- Table 3: page 662 in your text.
- http://davidmlane.com/hyperstat/z_table.html

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Comments

- The book suggests a continuity correction factor.
  - Add 0.5 to X for the probability of being less than or equal to X.
  - Subtract 0.5 from X for the probability of being greater than or equal to X.

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Continuity Correction Factor

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{\mathbf{360.5 - 380}}{\mathbf{15.35}} = \frac{\mathbf{-19.5}}{\mathbf{15.35}} \mathbf{-1.27}$$

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